TESTING FOR MULTIPLE BUBBLES WITH DAILY DATA

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Abstract

A new methodology for testing and dating economic bubbles based on a sign test with recursive median adjustment is presented. The methodology, originally proposed by Soo and Shin (2001) to detect random walks, is well-suited, theoretically, to deal with the many features of high-frequency financial time series such as leptokurtosis, conditional heteroskedasticity and heavy tails. The approach is very pragmatic and relies upon an analysis of the integration order of the analyzed series. This paper presents an application of the method to the North American stock market and the findings concerning the origination and collapsing of dates for the bubbles are consistent with those identified through the application of the previous theoretical literature.

JEL codes: G01, G14, C22, C15, C18, C58.

Key Words: bubbles, random walks, sign test.

Introduction

Identifying explosive bubbles that are characterized by having periodic collapses over time is an important concern in the current academic economic literature and it is also of great importance for practitioners. As argued by financial historians, financial crises are frequently preceded by an asset bubble or by dramatic credit booms (Ahamed, 2009; Fergunson, 2008; Marichall, 2010).

For that reason, in the aftermath of the period of the global financial crisis (2007-2009), and with the ongoing European crisis, central banks are focusing much of their efforts in developing methodologies to identify "excessive credit creation" or "excessive growth in assets prices"; however, clarification of the meaning of "excessive" in this context is not an easy task to accomplish. It is rooted in the very depths of economic theory that asks: when do prices (if ever) incorporate non-fundamental components in their dynamics? And, how can these periods of over- or under-reaction be detected?

Based on the postulates of the Efficient Market Hypothesis (EMH), many economists have declared the task to be impossible and that it is even imprudent to seek to combat price bubbles. However, modern econometric techniques can offer alternative ways to access the phenomenon and can produce explicit quantitative measures concerning bubble formation periods. These measures go beyond *ex post* detection techniques and can become a forward-looking dating algorithm that is capable of assisting regulators in market monitoring by the means of early warning diagnostic tests (Phillips, et al., 2011).

This paper offers a contribution to the empirical debate over dating bubbles. It is not intended to address theoretical problems related to the EMH or the impossibility of incorporating bubbles into the great majority of General Equilibrium models under Rational Expectations (Tirole, 1982). The intention of the author is to develop a recursive statistic that is able to detect explosive behavior in stock prices, and, in this sense, to contribute to the design of practical tools for central banks or practitioners for dealing with this phenomenon.

The document is divided into eight sections that follow this introduction. In the first, a quick theoretical background on bubbles is offered for the purpose of providing context to the reader. The second section reviews the empirical literature related to the topic of bubble detection. The third section presents the econometric models used to identify bubbles. The fourth section constructs a "bubble index" that is used to date origination and collapsing dates for the detected bubbles. The fifth section explores Evans´ (1991) model in search of a plausible alternative hypothesis for the random walk. The sixth section explores size and power considerations for the proposed test. The seventh section reports the results from applying the proposed technique to the North American stock market and the final section offers concluding comments.

1. Bubbles: a quick theoretical background

Although highly interesting, bubbles are a rare topic in the academic world. Bubbles have proved to be extremely difficult to incorporate into traditional equilibrium models with agents governed by the Rational Expectations hypothesis -RE (Tirole, 1982).

In the current debate concerning the existence of bubble episodes, the main explanations may be classified into two categories: on the one hand, there is the "macro" view in which bubbles are explained as helping to fix the market problem of a shortage of value storage. On the other hand, there is a more "micro" orientation that deals with agents that misbehave, due either to an agency problem or a behavioral problem. There is also a third branch of the explanatory literature which develops an "irrational exuberance paradigm" that takes refuge in a sunspots-style explanation that avoids the problem.

In the "micro" style, the seminal work of Allen and Gale (2000) provides a model of bubbles that are an agency problem. Banks do not have full control over the actions carried out by the investors in the economy. Given that these investors do not have to face the full consequences of their actions, these investors have a large incentive to get involved in

excessively risky projects. Whenever negative payoffs occur, they can simply declare themselves bankrupt and leave the bank to face the losses.

The work by Abreu and Brunnermeier (2003) is categorized as being within the same micro style. Here, behavioral biases are said to lead to the occurrence of bubble episodes. The authors show how rational arbitrageurs can ride a bubble, even if they know that the market will eventually collapse. For this explanation to hold, market timing is crucial: it is the dispersion in exit strategies among rational arbitrageurs which make bubbles possible. Another crucial aspect for generating bubbles in this framework are "noisy traders" (irrational agents), which despite their small numbers, are still responsible for initial bubble inflation.

Since Samuelson (1958), it is known on the "macro" side that bubbles can exist in an OLG structure, and they can be good, in the sense than they solve a dynamic inefficiency in the economy. In recent attempts to deal with the issue, Farmer (2012) incorporates the notion of market sentiment into the analysis and introduces the notion of a confidence crash, as earlier noted by Shiller (2000), within a general equilibrium framework. Negative consequences of bubbles are explored over allocations of real factors in the economy. It is also found the work by Branch and Evans (2011), which explain the bubble formation process in the context of an endogenous learning mechanism.[2]

2. Review of the Empirical Literature

Over several decades, the literature reflects numerous attempts to provide econometric tests to date bubbles. Gurkaynak (2008) provides a comprehensive treatment of rational bubbles as an empirical subject. This author divides the tests for detecting bubbles into five categories: variance bounds tests, West´s two-steps test, intrinsic bubbles, integration/cointegration-based tests, and tests of bubbles as an unobserved variable.

The variance bounds tests exploit the fact that under rational expectations the difference between actual and expected dividends is not forecastable and has a zero mean. The variance of the observed prices is naturally bounded because the expected increments in the prices are uncorrelated with all current information, including the price itself. Therefore, the *ex post* rational price (without bubble episodes) should vary at least as much as the observed prices. Observed prices are based on expected dividends and do not have the variation introduced by future forecasted errors, which are included in the *ex post* price. If the variance bound is violated by the data, this will be evidence that equity prices do not follow the fundamental pricing equation without bubbles.

Given that such a difference is unobservable and a terminal value for the price series must be assumed in practice, complications arise in the implementation of this idea. These complications can invalidate the results of the test which opens the door to the possibility of rejecting the fundamental pricing equation without having to accept the hypothesis of

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[2] The work by Carvalho et al. (2012) can be seen as an application of the Samuelson´s original idea with some nice extensions.

bubbles. The seminal work in this category has been done by Shiller (1981), Grossman and Shiller (1981), and LeRoy and Porter (1981). Nevertheless, the original methodologies were not intended to detect bubbles. This was a subsequent application by differing authors, such as Tirole (1985) or Blanchard and Watson (1982). Additional criticism of the methodology of variance bounds from that previously discussed is provided by Flavin (1983), Kelidon (1986) and more recently by Akdeniz et al. (2006).

The second category of West's two-steps test, concerns a two-step algorithm that explicitly considers the bubble possibility under the alternative hypothesis (West, 1987). In the first step, a model specification test is developed to detect bubbles, provided that misspecification is ruled out.

Along general lines, with the absence of bubbles, the Euler equation can be estimated in such a way that feasibly recovers the discount rate. Then, if dividends can be represented as an autoregressive process (AR), knowing the discount rate and the parameters of the AR process provides enough information to pin down the relationship between dividends and the fundamental market stock price. On the other hand, the actual relationship between stock prices and dividends can also be directly estimated by using a regression of stock prices on dividends. Under the null hypothesis of a non-bubble situation, the actual relationship should not differ from the constructed one.

This procedure makes it possible to trace a discrepancy which may arise to model misspecification or bubbles. Although it is theoretically appealing, its implementation is problematic (Gurkaynak, 2008). Specifically, the test is suited to detect only the kind of bubbles that show a non-zero correlation with dividends. The algorithm is poorly equipped to detect bubbles that are not correlated to the dividend process. Moreover, with the question about the interpretation of a non-bubble hypothesis, eventual rejection is still valid (Hamilton and Whiteman, 1985; Flood and Hodrick, 1986). Lastly, Flood et al. (1994) point out a different problem related to the estimation of the Euler equation. In theory, it should hold for any two periods, even for those infinitely apart, which seems to be a bad approximation of reality.

The intrinsic bubbles category explores a different type of bubbles, which may or may not be correlated with fundamentals. In the context of this kind of process, Froot and Obstfeld (1991) show that the nature of intrinsic bubbles imposes some non-linearity's on the price/dividend ratio, which otherwise would be linear. If non-linearity is found in the data, they are supposed to be there as a consequence of the over-reaction in prices to changes in the dividends that are due to the bubble. However, other sources of non-linearity´s as regime switching fundamentals cannot be disregarded. This was highlighted by Driffill and Sola (1998), Van Norden and Schaller (1993, 1999) and Schaller and Van Norden (2002).

The fourth group based on integration/co-integration approaches was developed mainly by Diba and Grossman (1987, 1988). The general idea is to impose more theoretical structure on the shape of bubbles. In this sense, the authors argue that non-arbitrage laws guarantee that if a bubble exists it should have existed from the beginning of trading operations;

otherwise, the agents would not be fully rational. Thus, the absence of bubbles will imply specific degrees of stationarity in dividends and stock prices, and explicit co-integration relationships between them. The presence of bubbles breaks down this relationship and provides an intuitive way to test for their presence in the data. That is, to test if the prices and dividends series are stationary, differentiating the number of times required to make them I (0), and to search for co-integrating relationships between these series.

Diba and Grosmman's approach suffers from the same problems as does every test for unit root or co-integration. On top of this, Evans (1991) has pointed out that a non-monotonically increasing bubble which never collapses to zero, but collapses to a number slightly above the zero bound, cannot be detected by this kind of technique. This methodology heavily rests on the non-stationary behavior of the series involved, which is not appropriated to a model of periodically collapsing bubbles. Some models of regime-switching have been developed to overcome Evans' criticism (Hall et al., 1999; Hall and Sola, 1993; Van Norde, 1996; Van Norden and Vigfusson, 1998). These models have emphasized how difficult is it to disentangle bubble's behavior from switching fundamentals. Recently, Phillips et al. (2012) and Phillips and Yu (2011) have expanded this approach of testing for bubbles and they have developed forward recursive regressions with iterative unit root tests to date multiple bubbles that collapse periodically.

Lastly, Wu (1997) models the bubble as an unobserved component by using a Kalman's filter approach. The approach makes an issue very clear that is related to all methodologies generally employed to measure bubbles as a residual: bubbles will create a residual, but any other misspecification of the model will do so, too. .

3. The Econometric Test

The discussion above notes that the development of a technique to date periodically collapsing bubbles is very far from being a closed issue in the financial literature. This paper is in line with many recent studies focused on detecting bubbles with given statistical conditions in terms of integration that financial series prices should present, provided that they are driven by their fundamentals.

The methodology is based on a unit-root-sign-test developed by So and Shin (2001). This test is particularly suited to deal with daily financial time series, given some stylized facts that are well documented in this branch of the literature, such as conditional heteroskedasticity, non-linear behavior, and leptokurtosis.

Following Ljungqvist and Sargent´s (2000) assumption of risk-neutral agents, the solution to the optimization problem of rational agents in an exchange economy is given by:

$$E_t \beta (d_{t+1} + p_{t+1}) = p_t \qquad [1]$$

Where the constant discount rate of the agents is β; d_{t+1} are the real payments for holding assent (dividends), and p_t are the (share) assets prices. Equation 1 states that, adjusted for dividends, the price follows a first-order Markov process and no other Granger variables

cause the prices. The solution to the stochastic difference equation in [1], has the following class of solutions:

$$p_t = E_t \sum_{j=1}^{\infty} \beta^j d_{t+j} + \xi_t \left(\frac{1}{\beta}\right)^t \qquad , \qquad [2]$$

Where ξ_t is any random process that obeys $E_t \xi_{t+1} = \xi_t$ (that is, ξ_t is a "martingale"). Equation 2 expresses the p_t price as the sum of discounted expected future dividends and a "bubble term" unrelated to any fundamentals.

Assuming a non-bubble situation leads to:

$$E_{t-1} p_t = p_{t-1} - E_{t-1} d_t \qquad [3]$$

That is, adjusted for dividends, in a non-bubble situation the price follows a random walk. Otherwise, if explosive behavior is detected in the data it can be considered as a signal of bubbles. Indeed, this is what EMH establishes: the markets will be efficient in the weak sense if the prices follow a random walk[3].

Nevertheless, to perform a test for random walk behavior with high frequency data is not a trivial task. Traditional tests such as Dickey-Fuller and Augmented-Dickey-Fuller do not seem appropriate in this context, given the assumptions about normality required to calculate the critical values in these cases. Leptokurtic and heavy-tailed series make the normality assumption not very appropriate for this environment.

So and Shin (2001) propose a unit root test based on sign operators and recursive median adjustments. The test has been shown to be invariant to conditional heteroskedasticity, monotonically increasing transformations of the data, heavy tails and outliers.

Set:

$$\bar{p}_t = h(p_t) \qquad [4]$$

$$p_t = \gamma(p_{t-1}, \dots, p_{t-k}) + u_t, t = 1, \dots n. \qquad [5]$$

where $\{\bar{p}_t\}$, $t = 0, \dots n$ is a set of observations, $h(p_t)$ is an unknown monotonic transformation of p_t, $\gamma(p_{t-1}, \dots, p_{t-k})$ is an unknown regression of interest, k is a positive interger, and $\{u_t\}$ is a sequence of random errors with conditional median equal to zero.

The main interest is in testing the null hypothesis of the random walk, which is given by:

$$H_0: p_t = p_{t-1} + u_t, t \geq 1 \qquad [6]$$

Against linear and non-linear explosive alternatives, under assumptions A1 and A2:

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[3] Following Campbell et al. (1997) in the typology of efficiency testing, this corresponds to the case of weak efficiency under random walk type 3.This is a very restricted case of market efficiency, but it is also the one most commonly used in the literature.

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A1. $\{sign(u_t)\}$ is a martingale difference sequence with respect to an increasing sequence of σ-fields $\{F_t\}$ such that $E[sign(u_t)|F_{t-1}] = 0$, $t = 1, ..., n$.

A2. $P[u_t|F_{t-1}] = 0$

So and Shin construct a sign test defined such that, if:

$$S_n(\gamma) = \sum_{t=1}^n signo(\bar{p}_t - \bar{p}_{t-1})signo(\bar{p}_{t-1} - \hat{m}_{t-1}) \geq n - 2B_n(\alpha) \qquad [7]$$

then the null hypothesis of random walk in [6] is rejected.

Here, $sign(\cdot)$ is the sign of u_t, defined as $sign(u_t) = 1$ if $u_t > 0$, $sign(0) = 0$ and $sign(u_t) = -1$ if $u_t < 0$; $\hat{m}_{t-1}$ is the median of $\{\bar{p}_t\}_{t=0}^n$; $B_n(\alpha)$ denotes the α-th lower quantiles of the binomial distribution with parameters $c(n, 1/2)$ $\gamma = 1$ and it represents an autoregressive process of order 1.

This random walks test form, against an alternative explosive hypothesis, follows the suggestion made by Diba and Grossman (1988) for conducing right-tailed unit root tests on the asset prices to detect the existence of bubbles.

However, as before mentioned, Evans (1991) demonstrated that this conventional procedure is not able to detect explosive bubbles when they manifest periodically collapsing behavior in the sample. Phillips et al. (2012) emphasize one way to overcome these criticisms is by using subsamples of the data to perform the tests in a forward recursive fashion. This methodology has the additional advantage of being able to establish the origination and collapsing dates for the bubbles detected, if any.

There are many ways to perform the sub sampling strategy, two of which are: rolling windows or recursive methods with fixed or flexible original sample lengths. The first one implies either the imposition of an optimal criterion for the size of the windows or the implementation of the test with several different window lengths to check robustness. The second procedure requires the determination of the starting sample size.

It is apparent that recursive methods by construction will manifest a delay in the identified origination and collapsing dates as a consequence of the inertia necessarily associated with them. Consequently, the first route is followed here, while the second one is extensively studied in the context of the Generalized Supremum ADF test developed by Phillips et al. (2012).

A final part of the methodology consists in applying the sign test not only to the series of prices, but also to the observable fundamentals (dividends). If explosive behavior is found in the prices and not in the fundamentals, it must be bubbles that are responsible for those findings.

4. The Bubble Index

In this section provides a simple index to date origination and collapsing times of multiple bubbles. A bubble index is defined such that:

$$BI_t = \begin{cases} S_{nt} - cv & \text{if } S_{nt} > cv \\ 0 & \text{otherwise} \end{cases} \qquad \forall\, t = 0,1,2,\dots,N-l \qquad [8]$$

Where, BI_t is the *bubble index* at period t, N is the sample size and l is the size of the rolling window. Quantity $cv := [n - 2B_n(\alpha)]$ is the lower α-th percentile of the binomial distribution with parameters of $(l, 1/2)$. S_{nt} is the test statistic defined in [7] at t, with $\gamma = 1$. Thus, origination and collapsing times of each bubble are given by:

$$t_B^i = \{t_j | BI_t > 0\} \qquad [9]$$

and

$$t_E^i = \{t_k | BI_t = 0, t_k > t_j\} \qquad [10]$$

where t_B^i is the period t_j that marks the beginning of the bubble, t_E^i is the period t_k that marks the end of the bubble, t_k and t_j are natural numbers such that $t_j, t_k \in t$; and i is a natural number that is used to identify the bubble. Note that depending on the size of the sample, the procedure can be used for dating multiple bubbles (if any).

As a practical consideration for avoiding trivial bubble detections, in the empirical application of this paper: an "origination point" of a bubble is marked as such only if the subsequent 20 data points in the bubble index also are different from zero. Analogously, to be marked as a collapsing period, a candidate date must fulfill the requirement that the following 20 points of the bubble index are also equal to zero.

5. The Evans' Model

As mentioned above, Evans (1991) highlighted a number of criticisms of traditional approaches for testing bubbles, such as those of Diba and Grossman (1988) and Hamilton and Whiteman (1985). In general, he postulates that these traditional tests for rational bubbles that are based on the analysis of the integration order of the financial times series of prices and its observable fundamentals can lead to misleading conclusions. One side unit root statistics, autocorrelation patterns and co-integration relationships perform poorly when detecting rational bubbles that collapse periodically within the sample with a relatively high probability of collapsing (above 10%).

Consequently, Phillips et al. (2012) propose a supremun ADF test that is repeatedly implemented on a forward expanding sampling sequence to detect Evans' "semi-stationary" bubbles. They compared their statistic against such an alternative hypothesis. This paper

proposes a different response as it considers a recursively adjusted sign test implemented in rolling windows of optimal length in terms of size and power criteria. Regardless, the Evans' model will be used as an alternative hypothesis for the calculation of the power.

In the Evans' world, simulated asset prices consists of a fundamental market component P_t^f and a bubble component of B_t. The fundamental part combines a random walk dividend process [11] and a fundamental pricing equation in [12]:

$$d_t = \mu + d_{t-1} + \varepsilon_{dt}, \; \varepsilon_{dt} \sim N(0, \sigma_d^2) \tag{11}$$

$$P_t^f = \sum_{j=1}^{\infty} (1+r)^{-j} \mathrm{E}_t \left(d_{t+j} \right) \tag{12}$$

One can solve [12] recursively to get the fundamental price component:

$$P_t^f = \frac{\mu \rho}{(1-\rho)^2} + \frac{\rho}{(1-\rho)} d_t \tag{13}$$

Evans' bubble component is given by:

$$B_{t+1} = \rho^{-1} B_t \varepsilon_{B,t+1}, \quad \text{if} \quad B_t \leq \alpha \tag{14}$$

$$B_{t+1} = [\delta + (\pi\rho)^{-1} \theta_{t+1}(B_t - \rho\delta)]\varepsilon_{B,t+1}, \quad \text{if} \quad B_t > \alpha \tag{15}$$

This series has the submartingale property, $E_t(B_t) = \rho^{-1} B_t$. Parameter μ is the drift of the dividend property, σ_d^2 is the variance of the dividend, $\rho^{-1} = 1 + r_f > 1$, here r_f is the discount factor which is constant. $\varepsilon_{B,t+1} = \exp(y_t - \tau^2/2)$ with $y_t \sim NID(0, \tau^2)$. δ and α are positive parameters with $0 < \delta < \rho^{-1}\alpha$, δ can be interpreted as the reinitializing parameter. θ_{t+1} is a Bernoulli process (independent of $\varepsilon_{B,t+1}$) which takes the value 1 with probability π and 0 with probability $(1 - \pi)$.

Equations [14] and [15] state that as long as $B_t \leq \alpha$ the bubble grows at a mean rate of ρ^{-1}. When eventually $B_t > \alpha$ the bubble "erupts" into a phase in which it grows at a faster rate$(\pi\rho)^{-1}$ as long as the eruption continues, but in which the bubble collapses with probability $(1 - \pi)$.

Finally the observed process of prices is defined as:

$$P_t = P_t^f + kB_t \tag{16}$$

Where $k > 0$ controls the relative magnitude of the fundamental and bubble components.

5.1. The Evans' Model in the data

The set of parameters required to perform the simulations in the next section is reported in Table 1.

Table 1. Parameter Settings

μ	σ_d^2	δ_0	ρ	α	B_0	π	ζ	τ	k
0.00032103	0.0006243	1	0.98	1	0.5	0.85	0.5	0.05	120

The parameters μ and σ_d^2 are set to correspond to the sample mean and the sample variance of the first daily difference in the S&P 500 stock price index dividend so that the settings are in accord with the empirical application presented in the next section.[4] The discount value ρ equals 0.98 (is allowed to vary from 0.98 to 0.995 in the size and power section). The parameter of the bubble weight k is set at 120 (again it is allowed to vary from 20 to 240). Other parameters are set in accord with Evans' original work and that by Phillips et al. (2012).

A sample trial of the Evans' model with the parameters in Table 1 is presented in Figure 1.

Figure 1

Simulated Evans' Model with daily parameters setting
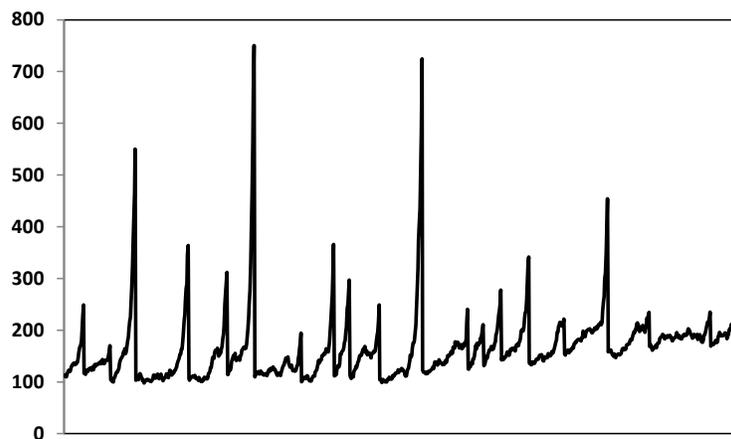
It is evident in this random trial that several collapsing bubbles are present, as expected from the Model.

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[4] To estimate μ and σ_d^2, the sample used was from Jan 3, 2000 to Oct 10, 2012, taking two considerations into account: first, data availability; and second, the fact that it is apparent from the visual inspection of the series that there is a break in the trend of the series around the beginning of the year 2000.

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6. Size and Power Simulations

This section discusses the size and power of the sign test, and therefore its ability to detect bubble behavior in the market. Several Monte Carlo simulations are conducted under different specifications of the model parameters.

Table 2 reports on the empirical sizes of the test using different α-th percentiles of the binomial distribution. The null hypothesis of the random walk is tested using 1000 simulations generated under the process described from [11] to [13] and by [16] with $k = 0$.

Table 2
Empirical sizes and size distortions

Empirical Sizes

α	T					
	500	1000	1500	2000	2500	3000
1	1.30	1.00	0.90	1.70	1.40	1.60
5	5.80	5.10	6.40	7.00	7.40	8.40
10	9.00	11.20	12.80	13.60	12.00	13.70

Size Distortions

α	T					
	500	1000	1500	2000	2500	3000
1	0.30	0.00	-0.10	0.70	0.40	0.60
5	0.80	0.10	1.40	2.00	2.40	3.40
10	-1.00	1.20	2.80	3.60	2.00	3.70

Note: The nominal sizes are reported in the first column. Empirical sizes for different sample lengths (T), from 500 to 3000 are reported in Columns from 2 to 7. 1000 simulations were used to construct the empirical sizes. The data generating process is given by equation [16] with $k = 0$.

As noticed in Table 2, size distortions are minimal in samples smaller than 1500 points. They become more pronounced in samples with larger amounts of data, which are located at the right side columns of the matrix.

Power calculations are computed using the Evans' Model in equations [11] to [16] as the alternative hypothesis, and they are reported in Table 3.

Table 3
Power of the sign test under different parameters specifications

ρ	T					
	500	**1000**	**1500**	**2000**	**2500**	**3000**
0.9800	68.1	73.2	79.3	79.6	81.2	84.1
0.9850	59.5	72.8	73.9	80.7	80.8	82.7
0.9900	45.1	58.6	67.4	71.1	71.9	76.9
0.9950	19.2	28.1	38.1	40.7	48.7	53.7

k	T					
	500	**1000**	**1500**	**2000**	**2500**	**3000**
20	55.9	67.0	70.7	74.8	78.2	80.3
60	66.6	71.5	75.8	74.4	80.4	79.6
90	68.1	74.9	78.8	80.2	80.9	82.6
120	68.1	73.2	79.3	79.6	81.2	84.1
240	68.4	79.7	85.1	85.8	86.9	85.3

Note: The power of the test under different parameter values is reported, as well as the power for different sample lengths (T), from 500 to 3000. 1000 simulations were used to construct the empirical sizes. The data generating process is given by equations [11] to [16] with αth lowest percentile given by 5, $k = 120$ and $ρ = 0.98$, unless it is specified in a different way.

The power of the test lies between 60 and 85 percent under sensible specifications of the parameters of the model, $ρ$ and k. The effect on the power of the quantity k is not very significant, even when the range is from 20 to 240. On the contrary, the effect of the parameter $ρ$ on the power is considerable. Values of $ρ$ very close to unity reduce the power of the test notable for all the studied sample lengths.

6.1.Optimal length of the rolling windows

A key step when working with rolling windows is determining the length of the window. There is no single criterion in the available literature for performing this task. Here, size and power considerations will be explored. For instance, from Table 2, in terms of power, it appears optimal to use windows as long as possible to estimate the sign statistic. Of course, the most extreme case would be working with a single window that includes the whole sample. However, from the discussion related to periodically collapsing bubbles and to the necessity of dating bubbles on a daily basis, a trade off arises. On the one hand, a window sensitive enough to new data to detect starting and collapsing times of bubbles as soon as possible is required. On the other hand, a long window associated with a lower probability of making type II statistical errors is desirable.

In Table 4, "gains" in terms of power from increasing sample size (length of the window) by 500 points are presented. As can be noticed, the bigger gains in terms of power are reported following the increments from 500 to 1000 points, and to some extent from 1000

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to 1500. This highlights the necessity of working with at least 1000 or 1500 points to make good use of the larger gains in power. The apparent results are that increasing the sample size from 2000 to 2500, or from 2500 to 2000, does not substantially increase the power of the test, unless the ρ parameter is very close to unity.

Table 4
Power "gains" from increasing the sample by 500 points

ρ	500-1000	1000-1500	1500-2000	2000-2500	2500-3000
0.9800	*7%*	8%	0%	2%	4%
0.9850	*22%*	2%	9%	0%	2%
0.9900	*30%*	15%	5%	1%	7%
0.9950	*46%*	36%	7%	20%	10%

k	500-1000	1000-1500	1500-2000	2000-2500	2500-3000
20	*20%*	6%	6%	5%	3%
60	*7%*	6%	-2%	8%	-1%
90	*10%*	5%	2%	1%	2%
120	7%	*8%*	0%	2%	4%
240	*17%*	7%	1%	1%	-2%

Note: Table 4 reports percentage increments in terms of power due to increasing the sample size. The largest increments in each case are highlighted with italics.

The suggestion for working with a window of 500 to 1500 data points is confirmed after analyzing empirical sizes reported in Table 2. The size distortion is lower for the shorter windows.

Following power considerations, the choice for the length of the window is 1000. Nevertheless, results that make use of 500 and 1500 data points are reported as a robustness exercise in the Appendix.

7. Testing for bubbles

The data is taken from Datastream International. Daily information was collected on the Standard and Poors 500 price index dividend yields. Optimally, this series should be deflected to make them real, but given the frequency of the data nominal series are used instead. The period of analysis is from January 1, 1990 to December 31, 2012, and therefore the working sample runs from March 3, 1986 to December 31, 2012. It compromises 7001 observations.

Figure 2 plots the time series trajectories of the S&P500 index. As can be seen, there are several periods that are candidates for being considered as bubbles.

Figure 2

**SP500 from Jan 01, 1990 to Dec 31, 2012**

Note: SP500, data taken from Datastream International

The sign statistic using rolling windows of 1000 observations is presented in Figure 3. The critical value (cv) reported corresponds to the 5th percentile of the binomial distribution with parameters (1000, ½).
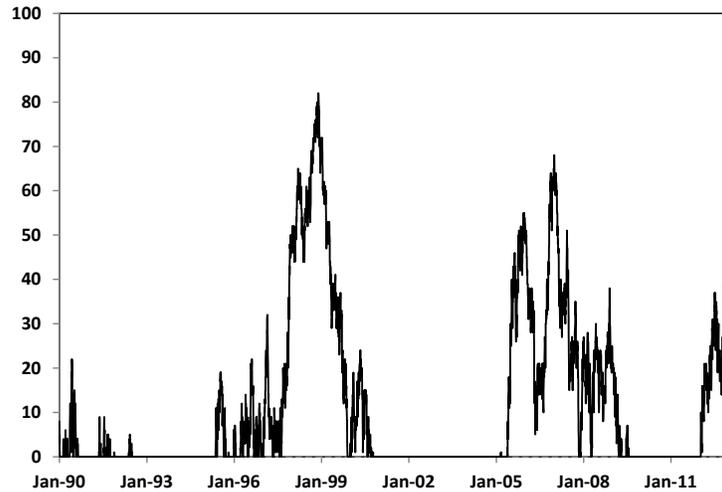
Figure 3

Sign-statistic with windows of 1000 observations

Note: Own elaboration

Testing for multiple bubbles with daily data
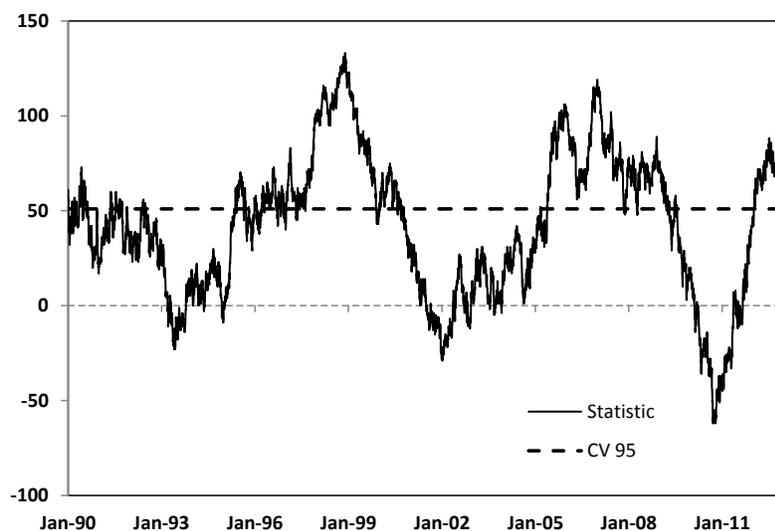
The Bubble Index is presented in Figure 4. Table 5 reports the origination and collapsing dates that arise from observation of the bubbles index.

**Figure 4**

Bubble Index

Note: own elaboration. The Index takes the value of zero when no bubble is detected at the chosen significance level.

Table 5 Origination and Collapsing dates

1000 POINTS WINDOW	
ORIGINATION	COLLAPSING
	4-Jan-1990
8-May-1990	21-Aug-1990
1-Jun-1995	15-Sep-1995
19-Mar-1996	22-Nov-1999
5-Jan-2000	9-Oct-2000
24-May-2005	20-Apr-2009
10-Jan-2012	

In order to discard explosive behavior in the observable fundamental dividend process that could potentially invalidate bubble detection, the sign test with recursive adjustments is applied to the series of dividends from the S&P500. The results are plotted in Figure 5.

Testing for multiple bubbles with daily data

Figure 5

Sign-statistic of the dividends series
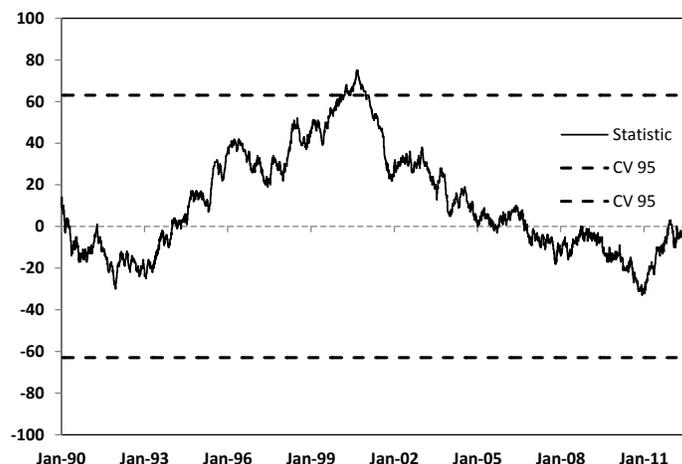
Note: cv are the critical values at 5%, under the explosive (+) and stationary alternative (-)

The random walk null hypothesis is rejected only from –Oct 4th, 1999 to Aug 24th, 2000. This behavior suggests that the bubble episode identified from Jan 5th 2000 to Oct 9th 2000 could be a fundamental reaction to the underlying fundamentals, making the rejection of the null theoretical hypothesis of non-bubble ambiguous. In this case, the following step would be to test for co-integration between the two explosive series; but this step will not be considered in this study.

The above analysis indicates that during recent decades, bubbles have been a notorious component for explaining the dynamics of the Standard and Poors 500. In particular, it is possible to identify three different periods that have been affected by bubbles: the middle of the 90s; the period from the immediate years preceding the financial crisis to the crisis itself; and the period at the end of the sample.

One tentative explanation for the origination of bubbles is provided by Caballero et al. (2008). These authors claimed that in a world with a scarce and asymmetric supply of financial assets, the United States financial markets are perceived as uniquely positioned to provide these instruments and therefore have been affected by a continuous (but irregular) flow of funds. In turn, these funds, together with low interest rates, have been responsible for the origination and collapse of periodic bubbles. The findings from this study are consistent with these theoretical claims.

Testing for multiple bubbles with daily data

8. Conclusions and limitations

The above results give new insight into a topic that is generally obscure in the financial literature by establishing a clear indication of bubbles in the dynamics of a stock price index. These results can be seen as a small step in the process of building an analytical framework to prevent bubbles through the means of monetary or exchange rate policies. Perhaps, accepting the possibility of bubbles and using econometric techniques to detect and measure them will expand the horizon of the role of central banks in the economy. For example, inflation targeting schemes could be enriched by the inclusion of financial assets prices into the monitored core of prices that are accessed by central bankers when making decisions on interest rates.

Nevertheless, the identification of bubbles with daily data is far from being a closed issue. In the implementation of the methodology described in this study, a set of assumptions have been made. These assumptions surely will (and should) be dispensed with in future research on the topic to gain a greater comprehension of the phenomenon. Particularly key assumptions are: i) the Evans' Model generates the bubbles that have been studied under the alternative hypothesis in the test of power; ii) the sign test used assumes that the process in equation 6 is of order 1, in case of a different order, the critical values in the test will suffer some modifications, some of which were explored by So and Shi (2001); iii) given the frequency of the data, it is not possible to work with series in real terms.

Six episodes of bubbles were identified in the data that were clustered in the middle of the nineties, during the turbulent period from 2004 to 2009 and at the end of the sample from the end of 2012 to the beginning of 2013.

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APPENDICES

Figure 6
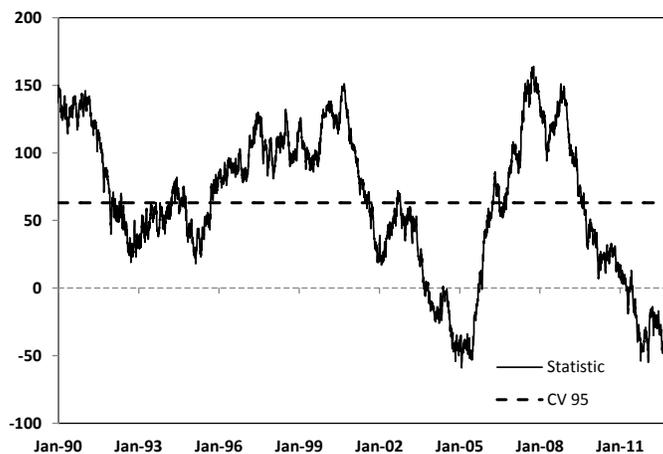
Sign-statistic with windows of 1500 observations
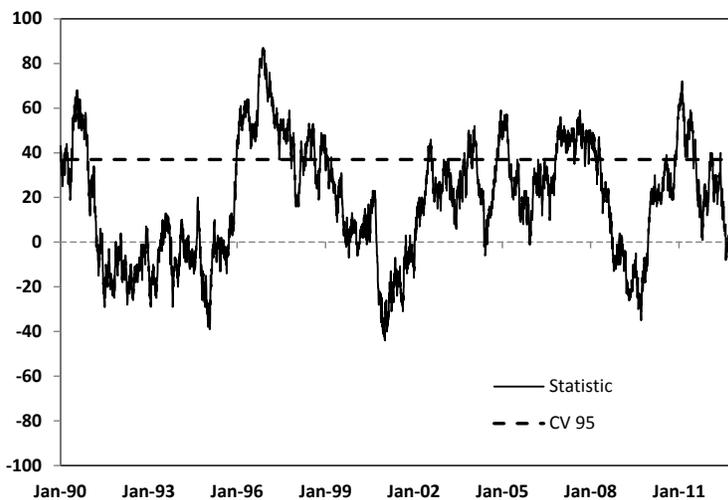

Figure 7

Sign-statistic with windows of 500 observations

Table 6
Origination and Collapsing dates 500 and 1500 windows

1500 POINTS WINDOW		500 POINTS WINDOW	
ORIGINATION	COLLAPSING	ORIGINATION	COLLAPSING
	5-Dec-1991		1-Jan-1990
31-Mar-1994	5-Oct-1994	29-May-1990	6-Dec-1990
11-Sep-1995	15-Aug-2001	25-Dec-1995	10-Dec-1997
6-Sep-2002	14-Oct-2002	30-Apr-1998	19-Aug-1998
4-Apr-2006	25-Jun-2006	27-Oct-1998	2-Feb-1999
29-Sep-2006	26-Aug-2009	25-Jun-2002	7-Aug-2002
		30-Oct-2003	20-Feb-2004
		25-Oct-2004	22-Mar-2005
		23-Oct-2006	30-Apr-2008
		2-Dec-2010	26-Jul-2011

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