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Law enforcement and drug trafficking networks: a simple model[1]

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Abstract

This article presents a theoretical model to explain the performance of illicit drug markets. The analytical framework is based on the oligopoly model of Poret and Téjedo (2006), but the latter is extended in a crucial respect: the influence of drug trafficking networks in the illicit drug markets is considered. The proposed model indicates that Poret and Téjedo were correct: the aggregate quantity of drugs sold is negatively affected by the intensity of the law enforcement policies applied and positively affected by the number of traffickers in the market. We also determined that the individual and aggregate sales in the market are positively affected by the network's average density. Our model is useful for explaining the failure of the *war against drugs* to halt the reproduction and expansion of illegal activities at a global level during the three past decades.

JEL classification: K42, D43, L13, C72, D85

Keywords: drug trafficking, illegal markets, law enforcement, social networks, game theory, oligopoly

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Law enforcement and drug trafficking networks: a simple model

1. Introduction

This article presents a theoretical model to explain the performance of illegal markets. The model can replicate some of the main forces underlying the trends of illegal drug markets and their prices. In other words, it is useful for explaining the prevalence and the expansion of drug trafficking activities during the past three decades, despite increasing efforts by the law enforcement authorities of producer and consumer countries in the *war against drugs*. This trend is one of the most striking facts in terms of the behaviour of illicit hard drug markets, namely, those for cocaine and heroin.

The model is based on the work of Poret and Téjedo (2006) and attempts to enrich theoretical understandings of illegal drug markets and law enforcement. Additionally, by examining the model's predictions, we endeavour to advance the solution of the referred puzzle. As in the work of Poret and Téjedo (2006), our starting premise is that an illegal drug market can be modelled as an oligopoly market structure. However, we extend their model in a crucial respect by considering the influence of drug trafficking networks in the illicit drug markets. We assume that the probability of detecting and arresting one trafficker depends on not his market share, but his share in his ego-network's sales. This assumption informs two basic features for the equilibrium solutions of the model. First, both the quantity of drugs sold by an individual trafficker and the aggregate sales in the market depend on the drug trafficking network's structure and density. Second, the effectiveness of law enforcement on reducing the drug supply depends on the characteristics of the drug trafficking network's structure.

In this paper, we analyse the aggregate impact of the drug trafficking network's structure on illegal activities. For this reason, we focus on solving the model for the simplest possible social structure: the case of *regular networks*. Future research should extend this analysis and the solution to irregular network's cases presented here, as greater algorithmic complexity is required to determine asymmetrical Nash equilibrium points. At the same time, more detailed insights concerning illegal goods and markets could be found. The rest of the paper is organised as follows. In the second part, the assumptions of the model are stated. In the third part, the model is solved in the case of regular networks. In the fourth part, key results for this case are presented. In the fifth part, two special cases for *regular networks* are examined. In the sixth part, an explanation for the ineffectiveness of the supply repression policies is provided in light of the model's main predictions. Finally, concluding remarks are offered.

2. A model of the drug market in the presence of drug trafficking networks

Let us assume, as in Poret and Téjedo's paper, that three different types of agents interact in the drug market: the traffickers, the drug law enforcement authorities and the buyers of drugs. There are $n \geq 1$ traffickers selling drugs in an oligopoly market structure. When exchanging q_i units of drugs, trafficker *i* completes q_i transactions, such that the sale of one unit of drugs equals the completion of one transaction.

The traffickers are embedded in a drug trafficking network, which binds them by social connections to other traffickers belonging to the same network. Formally, the network $\mathbf{g}$ is represented by the graph (N,V) comprising a set of nodes N and a set of links V between the nodes. The set N represents the set of *n* traffickers selling drugs in the market. When trafficker *i* knows *j* and has a social relation with him, then we set $g_{ij} = 1,$; otherwise, $g_{ij} = 0.$ In this model, $\mathbf{g}$ represents an undirected graph in which links are reciprocal, so $g_{ij} = g_{ji}, \forall i, \forall j \in N.$ Furthermore, we assume, following convention, that $g_{ii} = 0, \quad \forall i \in N.$

The complete network corresponds to the set of all subsets of *N* of size 2 and is denoted by $\mathbf{g}^c.$ In this special network, there is a link between *i* and *j* for each pair of traffickers. A network is empty if there are no links between traffickers, such that the latter behave as isolated agents, and $g_{ij} = 0, \forall i, \forall j \in N$. We can denote this case as $\mathbf{g}^e.$ Let $\mathrm{G} = \{\mathbf{g} \mid \mathbf{g} \subset \mathbf{g}^c\}$ be the set of all undirected networks on *N*. The network's structure is assumed to be known at a specific point in time.

Following Poret and Téjedo (2006) we assume that the law enforcement authorities have two different tools for thwarting trafficking activity. These tools are the authorities' efforts to apprehend traffickers and a sanction paid by traffickers when arrested and convicted. The first tool is expressed by the *probability of detection and arrest* faced by trafficker *i*, $h_i.$ To understand this expression, let us consider first, the probability that the law enforcement authorities detect a single transaction. We denote this probability as $h,$ such that $0 < h < 1$: this probability is identical for each transaction and each trafficker. Thus, the probability that a trafficker *i* is detected and arrested depends on h and is given by:

$$h_i(q_i, \mathbf{q_{-i}}, h, \mathbf{g}) = \frac{hq_i}{q_i + \sum_{j \neq i}^{n} g_{ij} q_j} \tag{1}$$

where $\mathbf{q_{-i}} \equiv (q_1, q_2, ... q_{i-1}, q_{i+1}, ..., q_n)$ denotes the vector of quantities of drugs sold by the other *j* traffickers. It is clear from this expression that the probability of detection and arrest for seller *i* depends positively on his own sales and inversely on the sales of the other *j* traffickers linked to him[3].

[3] In another paper (Raffo, in press), we present a related though different game-theoretic model with networks also based on Poret and Téjedo's analytical framework, but assuming a somewhat different form of the *probability of detection and arrest* more congruent with a well-known hypothesis in the field of criminology. It is notable that the main results of that model are consistent with those of the model presented here.

The form of this probability merits explanation because of its central role in this scenario. As in Poret and Téjedo's paper, we assume that this probability depends on *h* and on the visibility of the trafficker for law enforcement authorities. However, we developed an alternate way to capture this visibility in which the drug trafficking network is a key factor. The probability of detection and arrest of *i* depends positively on the quantity of drugs sold by the trafficker and inversely on the quantity of drugs sold by the other traffickers connected to him, i.e., his neighbours. This probability is independent of the quantity of drugs sold by all the other traffickers in the market, but only from the quantity sold by his neighbours. This means that the functioning of a concrete network's structure at a specific moment in time is an essential feature for the actual values of detection probability for the different agents. The term $\sum_{j \neq i}^{n} g_{ij} q_j$ captures the local interactions of agent *i* with other agents and can be understood as a type of *peer effects* that influences the delinquent activities of traffickers. Therefore, in this model, the probability of detection and arrest for *i* does not depend on his market share, but from his share in his ego-network's sales. Formally, the reason for this dependence is that the visibility of one trafficker is unrelated to other traffickers indirectly linked to him[4].

The structure of the probability of detection and arrest (given by Eq. (1)) is justified by the following reasons:

1) In the real world, the trafficking of illegal drugs depends on the operation of many drug trafficking networks. Hence, the visibility of one trafficker depends partly on his hierarchical position in, and his strategic importance to, the trafficking sub-network in which he is performing delinquent activities. The network's level of centrality of one seller captures this aspect of visibility.

2) Many studies on the sociology of crime (Hagedorn, 1988; Padilla, 1992; Thornberry *et al.* (2003)) and criminology (Sutherland, 1947) have highlighted the relevance of *peer effects* and local interactions in delinquent behaviour. More recently, work on the economics of crime has formally analysed the incidence of local externalities in delinquent activities in the context of network games. Hence, consistent with Calvó-Armengol and Zenou (2004) and Ballester *et al.* (2009)[5], we assume that the probability of detection and arrest of a trafficker reflects local complementarities in delinquency efforts, i.e., in drug trafficking efforts, across delinquents directly connected through the network.

3) The probability of detection and arrest of one seller does not depend on his share in the total market sales, only on his share in his ego-network's sales, because the

[4] However, a more realistic model should consider the impact of other traffickers indirectly linked to him, too.

[5] The work of Ballester *et al.* (2009) is based on the general model of network games with linear-quadratic utilities (see Ballester *et al.* (2006)) in which the Nash equilibrium actions of players are proportional to their respective centralities in the network in which they are embedded (Ballester *et al.*, 2006).

probability of his possible detection and arrest by the police or law enforcement authorities hinges on his direct links with other sellers. Hence, his visibility by law authorities depends on the weight of his sales in his ego-network sales; therefore, the probability of detection and arrest of an isolated agent who sells a certain quantity of drugs is not identical to that of a highly-connected agent who sells the same quantity. To discern this difference in sales, we can notice by Eq.(1) that the probability of detection and arrest for an isolated agent is always *h*, independent of his market share. This result is intuitive in that an isolated agent, by definition, has no links or relationships with other sellers, so law enforcement authorities are able to identify him directly without finding clues by examining the behaviour of other agents.

Let $N_i(\mathbf{g}) = \{ij \in \mathbf{g} \mid g_{ij} = 1\}$ be the neighbourhood of i in $\mathbf{g}$. Then, the probability of detection and arrest for *i* can be rewritten as

$$h_i(q_i, \mathbf{q_{-i}}, h, \mathbf{g}) = \frac{hq_i}{q_i + \sum_{j \in N_i(\mathbf{g})}^{n} q_j}$$

The other law enforcement tool used by the law authorities is a sanction paid by traffickers when arrested and convicted. We assume that this sanction is a linear function of the trafficked quantity (Burrus, 1999; Poret 2002 and Poret and Téjedo, 2006). This function is given by

$$S(q_i) = sq_i, \tag{2}$$

where *s* is a positive though finite parameter. Multiplying *s* by *h*, we obtain the expected unitary sanction for a trafficker: $e \equiv hs > 0$. The parameter *e* is measured on a real line. Jointly considering the two law enforcement tools, we can define the law enforcement costs faced by trafficker *i*. These costs are given by

$$C_i(q_i, \mathbf{q_{-i}}, e, \mathbf{g}) = \frac{eq_i^2}{q_i + \sum_{j \neq i}^{n} g_{ij} q_j}. \tag{3}$$

It can be proved that the law enforcement costs functions are increasing functions of q_i for all *i*, such that $\frac{\partial C_i(q_i, \mathbf{q_{-i}}, e, \mathbf{g})}{\partial q_i} \equiv MC_i(q_i, q_{-1}, g) > 0$. In fact,

$$MC_i(q_i, \mathbf{q_{-i}}, e, \mathbf{g}) = \frac{e \cdot \left[q_i^2 + 2q_i \cdot \left(\sum_{j \neq i} g_{ij} q_j \right) \right]}{\left(q_i + \sum_{j \neq i} g_{ij} q_j \right)^2} \tag{4}$$

Additionally, these functions are convex for all *i*:

$$\frac{\partial MC_i(q_i, \mathbf{q_{-i}}, e, \mathbf{g})}{\partial q_i} \geq 0 \tag{5}$$

The cost functions are strictly convex for all *i* linked at least to one other trafficker. It can be verified that

$$(\exists j \in N)(g_{ij} = 1) \Rightarrow \frac{\partial MC_i(q_i, \mathbf{q_{-i}}, e, \mathbf{g})}{\partial q_i} > 0.$$

Instead, for an isolated trafficker, the cost function is a lineal increasing function in q_i. In this special case, the law enforcement cost function is $C_i(q_i) = eq_i$, such that $MC_i(q_i, \mathbf{q_{-i}}, e, \mathbf{g}) = e$, and $\frac{\partial MC_i(q_i, \mathbf{q_{-i}}, e, \mathbf{g})}{\partial q_i} = 0.$

Furthermore, the marginal cost function of trafficker *i* is decreasing in the quantity of drugs sold by another trafficker *k* when *i* has a link with *k*. It can be confirmed that

$$\frac{\partial MC_i(q_i, \mathbf{q_{-i}}, e, \mathbf{g})}{\partial q_k} = \begin{cases} \dfrac{-2eq_i \cdot \left(\sum_{j \neq i} g_{ij} q_j\right)}{\left(q_i + \sum_{j \neq i} g_{ij} q_j\right)^3} < 0 & \text{if} \quad g_{ik} = 1 \\ 0 & \text{if} \quad g_{ik} = 0. \end{cases} \tag{6}$$

Eq. (6) shows that when there is a link between *i* and *k*, an increase in the quantity of drugs sold by trafficker *k* produces a positive externality, inducing a reduction in *i*'s marginal cost of law enforcement and necessarily, a reduction in the total cost.

For the sake of simplicity, we assume that the marginal cost to produce drugs (at a retail level) is constant and equals zero.

Finally, we assume that the demand of drugs faced by one trafficker is of the linear type $P(Q) = a - bQ,$ where *a* and *b* are positive parameters and $Q = \sum_{i=1}^{n} q_i.$ Nonetheless, for simplicity and without loss of generality we assume that *a= b=1,* so that

$$P(Q) = \begin{cases} 1 - Q & \text{if} \quad Q \in [0,1) \\ 0 & \text{if} \quad Q \in [1, \infty). \end{cases} \tag{7}$$

It is important to note that though the model is focused on the wholesale and retail drug-sale markets, it is useful for explaining the performance of the entire drug trafficking value chain. That is, we are assuming vertical integration of the remainder of the illegal sector based on two reasons. First, the entire value chain is complex, consisting of many stages, which encumbers developing a more realist but tractable model. Therefore, our assumption fits well for the sake of simplicity and tractability of the model. Second and more crucially, for most of the period of inquiry, i.e., during the eighties, the nineties and the first half of the following decade, the Colombian traffickers mostly controlled the large wholesale and retail distribution networks, thereby efficiently linking the production stages in producer countries with the distribution stages in consumer countries throughout the management of the transnational transport and trade of the illegal substances.

3. The solution of the model with regular networks

In what follows, we focus on the case of *regular networks*, in which all the traffickers on the network are linked to the same number of trafficking mates. This case is highly insightful for at least two reasons. First, the model can be solved easily by finding a symmetrical Nash equilibrium. Second, the aggregate impact of the network's average density at equilibrium can be easily analysed. Future papers should aim to solve the model in the general setting with different types of network structures, including cases of *irregular networks*. The solution of the model in the latter cases will capture the impact of the hierarchical positions and the levels of centrality of the different sellers in the network, thus corresponding to a more realist framework for real world trafficking networks. Nonetheless, in this work, for the sake of simplicity, we state and solve the model in the case of regular networks.

Let $d_i(\mathbf{g}) \leq n-1$ be the degree of agent i in $\mathbf{g}$. Then, assume that all the traffickers on the network are linked to the same number of traffickers, so $d_i(\mathbf{g}) = d, \ \ \forall i \in N,$ and hence, $\left|N_i\left(g\right)\right| = d, \ \ \forall i \in N.$ Given the symmetry in the network, d represents the average degree of the agents in the network, while $d/n-1$ represents the average density of the network. Although $d/n-1$ is a real valued variable, d is a discrete variable as n.

Consequently, the problem faced for each trafficker i is

$$\max_{q_i} \pi_i\left(q_i, \mathbf{q_{-i}}, e, \mathbf{g}\right) = \left(1 - Q\right) \cdot q_i - \frac{e q_i^{\ 2}}{q_i + \sum\limits_{j \in N_i(\mathbf{g})}^{n} q_j}, \tag{8}$$

The first-order condition is:

$$1 - 2q_i^* - \sum_{j \neq i} q_j - \frac{e \cdot \left(q_i^{*2} + 2q_i^* \sum\limits_{j \in N_i(\mathbf{g})} q_j \right)}{\left(q_i^* + \sum\limits_{j \in N_i(\mathbf{g})} q_j \right)^2} = 0, \quad i = 1,2,...,n. \tag{9}$$

It can be verified that for all i, the second-order condition for a maximum holds. The next proposition ensures that for each trafficker, the profit function is strictly concave. Denoting the vector $\left(q_1, q_2, ..., q_n\right)$ as $\mathbf{q}$, we can state the following proposition:

Proposition 1: $\forall i \in \{1,...,n\}$ *the profit function is a strict concave function of* q_i *for the region* $A = \left\{ \mathbf{q} \in \mathrm{R}^n \,\middle|\, q_i > 0 \wedge \sum_{i=1}^{n} q_i \leq 1 \right\}$.

Proof: *By deriving Eq. (8) with respect to* q_i *we obtain*

$$\frac{\partial \pi_i(q_i, \mathbf{q}_{-\mathbf{i}}, e, \mathbf{g})}{\partial q_i} = 1 - 2q_i - \sum_{j \neq i} q_j - \left[e \cdot \left(q_i^2 + 2q_i \sum_{j \in N_i(g)} q_j \right) \middle/ \left(q_i + \sum_{j \in N_i(g)} q_j \right)^2 \right].$$

Deriving again with respect to q_i*, yields:*

$$\frac{\partial^2 \pi_i(q_i, \mathbf{q}_{-\mathbf{i}}, e, \mathbf{g})}{\partial q_i^2} = -2 - \frac{\partial MC_i(q_i, \mathbf{q}_{-\mathbf{1}}, e, \mathbf{g})}{\partial q_i}.$$ *By Eq. (5) we know that* $\frac{\partial^2 \pi_i(q_i, \mathbf{q}_{-\mathbf{i}}, e, \mathbf{g})}{\partial q_i^2} < 0.$ ■

Due to the symmetry of the network, the impact of *i*'s neighbours on his costs is identical for each *i*, so the first order condition for each seller is identical. Thus, in turn, at equilibrium, $q_1^* = q_2^* = ... = q_n^*$. For this reason, the relevant solution for each agent is the symmetrical Nash equilibrium. Denoting the equilibrium level as $q^* = q_1^* = q_2^* = ... = q_n^*$, and considering that $|N_i(\mathbf{g})| = d, \forall i \in N,$ the simplified expression of the first order condition for *i* is

$$1 - 2q^* - (n-1)q^* - \frac{e(1+2d)}{(d+1)^2} = 0, \tag{9´}$$

The term $\frac{e(1+2d)}{(d+1)^2}$ in this expression corresponds to the equilibrium marginal law enforcement cost for *regular networks*. Solving for q^*, we obtain

$$q^*(n,e,d) = \frac{1}{(n+1)} \cdot \left[1 - \frac{e(1+2d)}{(d+1)^2} \right], \tag{10}$$

Thus, the equilibrium aggregate quantity of drugs sold in the market is

$$Q^*(n,e,d) = \frac{n}{(n+1)} \cdot \left[1 - \frac{e(1+2d)}{(d+1)^2} \right], \tag{11}$$

The solution given by Eq. (11) is the unique positive equilibrium aggregate quantity if and only if

$$e < \frac{(d+1)^2}{(1+2d)} \equiv \bar{e}. \tag{12}$$

Level $\bar{e}$ of the expected unitary sanction represents the threshold beyond which there is no illicit market with *n* traffickers. It can be verified that $\frac{(d+1)^2}{(1+2d)} \equiv \bar{e} > 1.$ Additionally, given that $0 \le d \le n-1$, Eq. (12) implies that $e < \frac{n^2}{(2n-1)}$. Because repression is costly, it is reasonable to suppose that Eq. (12) holds (Poret and Téjedo, 2006). Nevertheless, it is significant that in this model, the maximum law enforcement level is independent of the number of firms, as in Poret and Téjedo's paper; rather, it depends on the average degree of the drug trafficking networks. Indeed, it can be verified that $\bar{e}$ increases with *d*, such that the higher the average degree of the drug trafficking, the stronger the level of repression imposed by the enforcement authorities should be to completely suppress illicit market operations. Total suppression could not attained in practice because of the high cost of the implementation of repression policies. However, the model does not account for different costs, namely social costs, budget costs and financial costs, associated with such policies.

4. Some predictions of the model with regular networks

Eq. (11) shows that Proposition 1 of Poret and Téjedo's paper holds. The next proposition is stated under the assumptions of our model[6].

Proposition 2: *Assume that* $0 \le d \le n-1$ and $e < \bar{e}$. *Then,*
(i) Given a number of traffickers (n) and the network's average degree, an increase (decrease) in the law enforcement intensity causes a decrease (an increase) in the total quantity of illicit drugs sold.
(ii) Given the law enforcement intensity (e) and the network's average degree (d), the total quantity sold increases with the number of traffickers present in the market.

Proof:
(i) *The derivative of* Q^* *with respect to e gives*

$$\frac{\partial Q^*(n,e,d)}{\partial e} = -\frac{n}{(n+1)} \cdot \frac{(1+2d)}{(d+1)^2} < 0.$$

(ii) *From Eq. (11) the rate of change of* Q^* *with respect to one unitary change in n gives*

$$Q^*(n+1,e,d) - Q^*(n,e,d) \equiv \frac{\Delta Q^*(n,e,d)}{\Delta n} = \frac{1}{(n+1)(n+2)} \cdot \left[1 - \frac{e(1+2d)}{(d+1)^2}\right];$$

as $e < \frac{(d+1)^2}{(1+2d)}$, *this is positive.* ■

[6] It is crucial to notice that Poret and Téjedo denote the aggregate quantity of drugs sold as *q*, whereas we denote this variable as *Q*. In our notation, q^* represents the equilibrium quantity of drugs sold by an individual seller.

The first part of Proposition 2 proves that *ceteris paribus* a tougher anti-drug law enforcement policy causes a reduction in the quantity of drugs sold. This effect is due to the impact of the increases of *e* on the quantity of drugs sold by each trafficker: a one-unit increase in *e ceteris paribus* increases the total and marginal cost of subjecting this trafficker's activity to law enforcement. From Eqs. (3) and (4), it can be verified that $\frac{\partial C_i(q_i,\mathbf{q}_{-\mathbf{i}},e,\mathbf{g})}{\partial e}=\frac{q_i^{\,2}}{q_i+\sum_{j\neq i}^{n} g_{ij}q_j}>0$ and $\frac{\partial MC_i(q_i,\mathbf{q}_{-\mathbf{i}},e,\mathbf{g})}{\partial e}=\frac{\left[q_i^{\,2}+2q_i\cdot\left(\sum_{j\neq i} g_{ij}q_j\right)\right]}{\left(q_i+\sum_{j\neq i} g_{ij}q_j\right)^2}>0.$ Consequently, at equilibrium, each trafficker tends to sell a lower quantity of drugs. For this reason, an increase in *e* causes a decrease in the equilibrium quantity sold by each trafficker and hence, in Q^*. In fact,

$$\frac{\partial q^*(n,e,d)}{\partial e}=-\frac{1}{(n+1)}\cdot\frac{(1+2d)}{(d+1)^2}<0.$$

The derivative $\frac{\partial Q^*(n,e,d)}{\partial e}=-\frac{n}{(n+1)}\cdot\frac{(1+2d)}{(d+1)^2}$ can be understood as the *marginal efficiency of law enforcement*. Although it is always negative, its actual value depends on the concrete network's structure (i.e., on the particular levels of *n* and *d*), so that the effectiveness of the law enforcement policy is not always the same. First, for a large (small) number of traffickers (nodes in the network), the marginal impact of the law enforcement over Q^* is stronger (weaker). This result can be explained by the fact that given the value of *d*, an increase in *e* affects more traffickers when *n* is large so that the aggregate fall in the sales is larger. It can be proved that

$$\frac{\partial Q^*(n+1,e,d)}{\partial e}-\frac{\partial Q^*(n,e,d)}{\partial e}\equiv\frac{\Delta}{\Delta n}\left(\frac{\partial Q^*(n,e,d)}{\partial e}\right)=-\frac{1}{(n+1)(n+2)}\cdot\frac{(1+2d)}{(d+1)^2}<0.$$

Second, when the network's average degree is high (low), a tougher anti-drug law enforcement policy causes a lower (larger) decrease in Q^*. This result occurs because within a denser network, the law enforcement costs and the marginal law enforcement costs faced by the traffickers are lower due to the presence of stronger positive externalities between them; hence, traffickers are able to sell drugs more efficiently[7]. The increase in the marginal cost induced by a marginal rise in *e* is smaller for high values of *d* because $\frac{\Delta}{\Delta d}\left(\frac{\partial MC_i}{\partial e}\right)<0.$[8] The impact of the average degree's level in the marginal efficiency of the

[7] See ahead the explanation of the impact of changes in *d* over Q^*.

[8] To prove this result is necessary to take into account the fact that with regular networks the marginal costs of *i* can be written simply as $CM_i(q_i,\mathbf{q}_{-\mathbf{i}},e,d)=\frac{eq_i^{\,2}}{q_i+d\cdot q_j}$.

law enforcement policy can be captured by the rate of change of $\frac{\partial Q^*(n,e,d)}{\partial e}$ when there is one unitary change in *d* equalling $\frac{\Delta}{\Delta d}\left(\frac{\partial Q^*(n,e,d)}{\partial e}\right)=\frac{n}{(n+1)}\cdot\left[\frac{2d(2+d)+1}{(1+d)^2(2+d)^2}\right]>0.$

Third, $\frac{\partial Q^*(n,e,d)}{\partial e}$ is unaffected by the level of *e*, so Q^* is a linear decreasing function of this variable.

The second part of Proposition 2 demonstrates that, *ceteris paribus*, an increase in the number of traffickers in the market causes a rise in the total quantity of drugs sold. As Poret and Téjedo (Op. Cit.) observe, this effect is *a competition effect*. Nonetheless, it is important to notice that an increase in *n* results in a decrease in the quantity of drugs sold by each individual trafficker such that $q^*(n+1,e,d)-q^*(n,e,d)=-\frac{1}{(n+1)(n+2)}\cdot\left[1-\frac{e(1+2d)}{(d+1)^2}\right]<0.$

The enhanced competition negatively impacts the market price due to the shape of the inverse demand function and, consequently, lowers the marginal income for each trafficker. Therefore, the latter effect is outweighed by the presence of additional sellers in the market.

As $\frac{\partial Q^*(n,e,d)}{\partial e}$, $Q^*(n+1,e,d)-Q^*(n,e,d)$ depends on the concrete network's structure as well as on the intensity of repression. First, it depends negatively on *e*: high (low) levels of repression diminish (increase) the positive impact of a marginal increase of *n* over Q^*. This result can be explained by the effect of a change in *e* over Q^*, as we proved by the first part of Proposition 2. It can be verified that $\frac{\Delta}{\Delta n}\left(\frac{\partial Q^*(n,e,d)}{\partial e}\right)=\frac{\partial}{\partial e}\left(\frac{\Delta Q^*(n,e,d)}{\Delta n}\right)<0.$

Second, it depends negatively on the number of traffickers. Thus, Q^* is a concave increasing function of *n*, meaning that the positive impact of a marginal increase in the traffickers on Q^* is lower (higher) for a large (small) number of them because the negative effect of competition on prices and individual sellers' marginal incomes become stronger than the direct impact of more sellers in the market. To demonstrate this change, it is helpful to note that the second rate of change of Q^* with respect to *n*, $\frac{\Delta}{\Delta n}\left(\frac{\Delta Q^*(n,e,d)}{\Delta n}\right)\equiv\frac{\Delta^2 Q^*(n,e,d)}{\Delta n^2}$, equals $-\frac{2}{(n+1)(n+2)(n+3)}\cdot\left[1-\frac{e(1+2d)}{(d+1)^2}\right]<0.$ At the limit, for a very large level of *n* ($n\to\infty$), Q^* tends towards $\left[1-\frac{e(1+2d)}{(d+1)^2}\right]$, and $\frac{\Delta Q^*(n,e,d)}{\Delta n}$ tends towards zero.

Third, it depends positively on *d* because with denser networks, an increase in *n* induces the generation of more positive externalities due to the presence of a greater number of connections between the sellers. The explanation is identical to that justifying the impact of d over $\frac{\partial Q^*}{\partial e}$. It can be seen that $\frac{\Delta^2 Q^*(n,e,d)}{\Delta n \Delta d} = \frac{e}{(n+1)(n+2)} \cdot \left[\frac{2d(2+d)+1}{(1+d)^2(2+d)^2} \right] > 0.$

Now, let us consider the impact of the changes in the average network density. Given a number of traffickers n, an increase (decrease) in the average density, $\frac{d}{(n-1)}$, is proportional to an increase (decrease) in the average degree, d. Consequently, for a given n, we can analyse the effect of changes in the network's density on sales by examining the impact of changes in d over them. We can therefore state the next proposition.

Proposition 3: *Assume that* $1 \le d \le n-1$ and that $e < \bar{e}$. *Then, given a number of traffickers (n) and a law enforcement intensity level (e), an increase (decrease) in the average density of the network causes an increase (decrease) in the total quantity of illicit drugs sold.*

Proof: *From Eq. (11), the rate of change of* Q^* *with respect to one unitary change in d gives*

$$Q^*(n,e,d+1) - Q^*(n,e,d) = \frac{\Delta Q^*(n,e,d)}{\Delta d} = \frac{ne}{(n+1)} \cdot \left[\frac{2d(2+d)+1}{(1+d)^2(2+d)^2} \right] > 0. \blacksquare$$

Proposition 3 captures the impact of the average density of the network on the illegal activities for given values of n and e. The intuition underlying this effect is the following: the greater number of social connections each trafficker has, the higher the quantity of drugs sold by him. It can be verified that

$$q^*(n,e,d+1) - q^*(n,e,d) \equiv \frac{\Delta q^*(n,e,d)}{\Delta d} = \frac{e}{(n+1)} \cdot \left[\frac{2d(2+d)+1}{(1+d)^2(2+d)^2} \right] > 0.$$

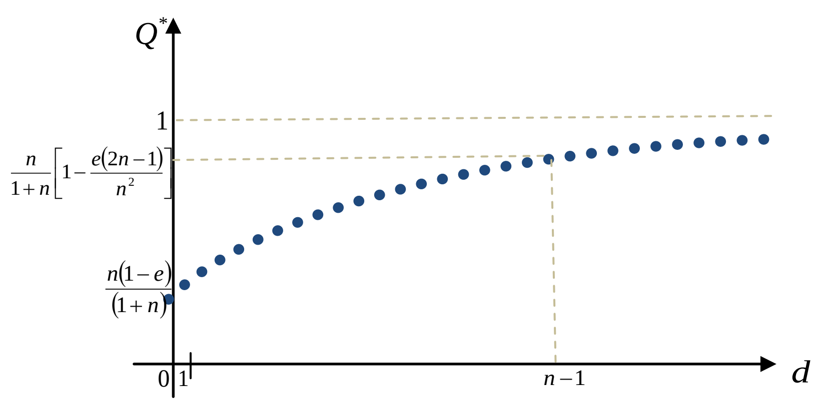
This effect is caused by the negative impact of the increase in d on i's law enforcement cost. Namely, due to the symmetry of the solution, it can be proved that, *ceteris paribus*, $q^* \quad \frac{\Delta C_i(q^*,d,e)}{\Delta d} = -\frac{e \cdot q^*}{(1+d)(1+2d)} < 0.$ In other words, the more links a seller has, the stronger the network's positive externalities he faces on account of his costs.

The rate of change of Q^* with respect to one unitary change in d depends on the concrete network structure and on the intensity of repression. First, it depends positively on e, for the reason that with higher (lower) levels of e, the negative impact of one marginal increase in d over the law enforcement costs is stronger (weaker). This impact can be seen directly above from $\frac{\Delta C_i(q^*,d,e)}{\Delta d} = -\frac{e \cdot q^*}{(1+d)(1+2d)} < 0$. The second rate of change of Q^* with respect to *e* equals $\frac{\Delta}{\Delta d}\left(\frac{\partial Q^*(n,e,d)}{\partial e}\right) \equiv \frac{\partial}{\partial e}\left(\frac{\Delta Q^*(n,e,d)}{\Delta d}\right)$, which we proved is positive.

Second, it depends positively on *n* because $\frac{\Delta^2 Q^*(n,e,d)}{\Delta d \Delta n} = \frac{e}{(n+1)(n+2)} \cdot \left[\frac{2d(2+d)+1}{(1+d)^2(2+d)^2}\right] > 0$. With a large (small) number of traffickers, one marginal increase in *d* induces the generation of a greater number (or fewer) positive externalities due to the presence of more nodes in the network.

Third, for all $d \geq 1$, $\frac{\Delta Q^*(n,e,d)}{\Delta d}$ depends negatively on *d*, so Q^* is a concave increasing function of *d*. It can be verified that $\frac{\Delta^2 Q^*(n,e,d)}{\Delta d^2} < 0$. Consequently, Q^* is a concave function of *d*, so with a high (low) network's average degree, the marginal impact of one increase in *d* tends to be smaller (larger). This effect suggests that the enhanced competition caused by one increase in *d* also negatively impacts prices, thus tending to weaken the positive impact the enhanced competition has over Q^*. The higher the level of *d*, the stronger the negative impact of the enhanced competition over prices and, hence, the lower the level $\frac{\Delta Q^*(n,e,d)}{\Delta d}$. Furthermore, for a high level of *d* (i.e. $d \to (n-1)$) ceteris paribus n, Q^* is asymptotic to $\frac{n}{1+n}\left[1 - \frac{e(2n-1)}{n^2}\right]$, and $\frac{\Delta Q^*(n,e,d)}{\Delta d}$ tends towards $\frac{e}{(1+n)^3}\left[\frac{2(n-1)(n+1)+1}{n}\right]$. For very high levels of *n* ($n \to \infty$) Q^* is asymptotic to 1 as d tends likewise to be very large $(d \to \infty)$. The following figure plots the equilibrium aggregate quantity of drugs sold as a function of *d*:

FIGURE N° 1
EQUILIBRIUM QUANTITY OF DRUGS AS A FUNCTION OF *d*[9]



Source: prepared by author.

[9] For an improved understanding of Figure N ° 1, we represent graphically the values of *d* in a different scale of the values of Q^*. The same holds for Figure 2.

Replacing Q^* in the inverse demand function, we obtain

$$P^*(Q^*) = 1 - \frac{n}{(n+1)} \cdot \left[1 - \frac{e(1+2d)}{(d+1)^2} \right]. \tag{13}$$

Eq. (13) is only meaningful for positive values of P^*. Actually, we always obtain $P^* > 0$. It can be proved that under the assumptions of the model, $1 > \frac{n}{(n+1)} \cdot \left[1 - \frac{e(1+2d)}{(d+1)^2} \right]$, which is equivalent to stating that $Q^* < 1$. This equation proves that at equilibrium, the price of illegal drugs increases as the intensity of repression, *e*, grows. This outcome results from the effects of increases in *e* over Q^* and is supported by numerous works in the field[10]: this is one of the most important results in *the economics of crime* and in *the theory of illegal goods*. The next proposition states this formally, along with another important assertion.

Proposition 4: *Assume that* $1 \le d \le n-1$ and that $e < \bar{e}$. *Then,*
(i) Given a number of traffickers (n) and the network's average degree (d), an increase (decrease) in the law enforcement intensity causes an increase (a decrease) in the equilibrium market price of illegal drugs.
(ii) Given the law enforcement intensity level (e) and the network's average degree (d), the equilibrium price of illegal drugs decreases as the number of traffickers in the market increases.

Proof:
(i) *Deriving* P^* *with respect to e gives*

$$\frac{\partial P^*(n,e,d)}{\partial e} = \frac{n}{(n+1)} \cdot \frac{(1+2d)}{(d+1)^2} > 0.$$

(ii) *From Eq. (13) the rate of change of* P^* *with respect to one unitary change in n gives*

$$P^*(n+1,e,d) - P^*(n,e,d) \equiv \frac{\Delta P^*(n,e,d)}{\Delta n} = -\frac{1}{(n+1)(n+2)} \cdot \left[1 - \frac{e(1+2d)}{(d+1)^2} \right],$$

which is moreover negative as $e < \frac{(d+1)^2}{(1+2d)}$. ■

The derivative $\frac{\partial P^*(n,e,d)}{\partial e}$ can be conceptualised as the *marginal impact of law enforcement over drug prices*. It is useful to notice that it depends on the network's structure. First, note that it depends positively on the number of traffickers in the network;

[10] See Poret (2002), Ortiz (2002, 2003, 2009), Miron (2001), Poret and Téjedo (2006), Becker, Murphy and Grossman (2006) and Caulkins and Reuter (2010), among many other works.

it can be verified that $\frac{\partial P^*(n+1,e,d)}{\partial e}-\frac{\partial P^*(n,e,d)}{\partial e}\equiv\frac{\Delta}{\Delta n}\left(\frac{\partial P^*(n,e,d)}{\partial e}\right)=\frac{1}{(n+1)(n+2)}\cdot\frac{(1+2d)}{(d+1)^2}>0.$ This influence is explained by the impact of *n* over $\frac{\partial Q^*(n,e,d)}{\partial e}$, as previously explained. Second, it depends negatively on *d* because higher values of the network's average degree weaken the marginal impact of law enforcement over Q^* , as elucidated before, in turn causing a lower increase in drug prices. In fact, $\frac{\Delta}{\Delta d}\left(\frac{\partial P^*(n,e,d)}{\partial e}\right)=-\frac{n}{(n+1)}\cdot\left[\frac{2d(2+d)+1}{(1+d)^2(2+d)^2}\right]<0$ corresponds to the negative value of $\frac{\Delta}{\Delta d}\left(\frac{\partial Q^*(n,e,d)}{\partial e}\right)$. Additionally, it is important to note that $\frac{\partial P^*(n,e,d)}{\partial e}$ is independent of *e*, so Q^* is a lineal function of *e*.

We have commented the meaning and relevance of part *i)* of Proposition 4. Now, consider part *ii)*. It shows that increased competition lowers the price of illegal drugs by inducing increases in the total quantity of drugs sold. The rate of change of P^* with respect to *n*, $\frac{\Delta P^*(n,e,d)}{\Delta n}=-\frac{1}{(n+1)(n+2)}\cdot\left[1-\frac{e(1+2d)}{(d+1)^2}\right]$, depends on the intensity of repression and the concrete network's structure.

First, it depends positively on *e*. As previously proven, high (low) levels of repression weaken (strengthen) the impact of a marginal increase of *n* over Q^*. Thus, high levels of *e* induce increases in the marginal impact of *n* over *Q* (decreases of it in absolute value terms). Indeed, $\frac{\partial}{\partial e}\left(\frac{\Delta P^*(n,e,d)}{\Delta n}\right)=-\frac{\partial}{\partial e}\left(\frac{\Delta Q^*(n,e,d)}{\Delta n}\right)=\frac{1}{(n+1)(n+2)}\cdot\frac{(1+2d)}{(d+1)^2}>0.$

Second, it depends positively on *n*. Consequently, P^* is a convex decreasing function of *n*. This result can be understood as consistent with the relationship between Q^* and *n* at equilibrium. Third, it depends negatively on *d* because $\frac{\Delta^2 P^*(n,e,d)}{\Delta n\Delta d}=-\frac{e}{(n+1)(n+2)}\cdot\left[\frac{2d(2+d)+1}{(1+d)^2(2+d)^2}\right]<0.$ This dependence holds for the positive impact of the network's average degree over $\frac{\Delta Q^*(n,e,d)}{\Delta n}$.

By the same token, an increase in the network's average degree and hence in the network's average density has a similar effect on P^* as an increase of *n*, *ceteris paribus*. The next proposition states this influence formally.

Proposition 5: *Assume that* $0\le d\le n-1$ and that $e<\bar{e}$. *Then, given a number of traffickers (n) and a law enforcement intensity (e), an increase (decrease) in the average* *density of the network causes a decrease (an increase) in the equilibrium price of illicit drugs.*

Proof:

From Eq. (13) the rate of change of P^* *with respect to one unitary change in d gives*

$$P^*(n,e,d+1)-P^*(n,e,d)\equiv\frac{\Delta P^*(n,e,d)}{\Delta d}=-\frac{ne}{(n+1)}\cdot\left[\frac{2d(2+d)+1}{(1+d)^2(2+d)^2}\right].$$

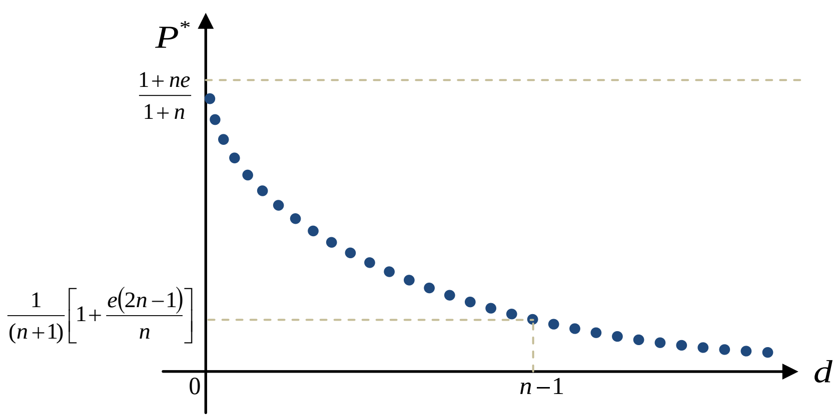
which is negative as $e<\frac{(d+1)^2}{(1+2d)}$. ■

The impact of an increase of *d* over P^* can be understood as a second *competition effect* because it enhances the traffickers' ability to sell illicit drugs, even with a fixed *n*. It also depends on the network's structure. First, it is enhanced with an increase in *e*, i.e. grows in absolute value as *e* increases. This relationship occurs because a marginal increase in *e* strengthens $\frac{\Delta Q^*(n,e,d)}{\Delta d}$. It can be seen that $\frac{\partial}{\partial e}\left(\frac{\Delta P^*(n,e,d)}{\Delta d}\right)=-\frac{n}{(n+1)}\cdot\left[\frac{2d(2+d)+1}{(1+d)^2(2+d)^2}\right]<0.$

Second, it also depends negatively on *n* because $\frac{\Delta^2 P^*(n,e,d)}{\Delta d\Delta n}=-\frac{e}{(n+1)(n+2)}\cdot\left[\frac{2d(2+d)+1}{(1+d)^2(2+d)^2}\right]<0.$

This dependence is due to the positive (negative) impact of larger (smaller) values of *n* over $\frac{\Delta Q^*(n,e,d)}{\Delta d}$, as previously explained. Third, it depends positively on *d*, a direct consequence of the concavity of Q^* as a function of this parameter. Thus, P^* is a convex decreasing function of *d*, as visualised in the following figure.

FIGURE N° 2
EQUILIBRIUM PRICE OF DRUGS AS A FUNCTION OF *d*



Source: prepared by author.

Finally, in solving the model with uniform networks, it is important to calculate the equilibrium profits of a seller. These are:

$$\pi^*(n,e,d) = q^*(n,e,d)\left(P^*(n,e,d) - \frac{e}{(1+d)}\right). \tag{13}$$

Using Eqs. (9) and (12), we obtain:

$$\pi^*(n,e,d) = \frac{1}{(n+1)}\cdot\left[1-\frac{e(1+2d)}{(d+1)^2}\right]\cdot\left\{1-\frac{n}{(n+1)}\cdot\left[1-\frac{e(1+2d)}{(d+1)^2}\right]-\frac{e}{(1+d)}\right\} \tag{14}$$

It can be proven that $\forall n \in \mathrm{N}_+$; this expression is positive when $e < \bar{e}$.

Now, let us consider the impact of the changes in *e, n* and *d* over $\pi^*(e,n,d)$. The impact of changes in *e* depends on *e*'*s* own level, the number of sellers, *n*, and the network's average degree. The following proposition states these dependences formally:

Proposition 6: *Assume the following:* $1 < d \le n-1$; $e < \bar{e}$ *; and* $n \ge 5$. *Then, given a number of traffickers (n) and the network's average degree (d), an increase (decrease) in the law enforcement intensity (e) causes an increase (a decrease) in the trafficker's equilibrium profits for relatively small (large) values of e.*

Proof**:** *Eq. (14) indicates that the derivative of* π^* *with respect to e gives*

$$\frac{\partial \pi^*(n,e,d)}{\partial e} = \frac{\partial q^*(n,e,d)}{\partial e}\cdot\left(P^*(n,e,d)-\frac{e}{(d+1)}\right)+q^*(n,e,d)\cdot\left(\frac{\partial p^*(n,e,d)}{\partial e}-\frac{1}{(1+d)}\right). \tag{15}$$

The first term of this expression is negative because $\frac{\partial q^*(n,e,d)}{\partial e} < o$ *, and* $\left(P^* - \frac{e}{(d+1)}\right) > 0$ *. By contrast, it can be proven that the sign of the second term is positive, except for the special case of isolated sellers with d=0.*

Let $\tilde{e} \equiv \bar{e}\cdot\frac{(d(n-3)-2)}{2(d(n-1)-1)}$ *denote the level of e for which* $\frac{\partial \pi^*(n,e,d)}{\partial e} = 0$. *Eqs. (10) and (13) can be used to prove that* $\frac{\partial \pi^*(n,e,d)}{\partial e} \overset{\ge}{\underset{<}{}} 0 \Leftrightarrow e \overset{\le}{\underset{>}{}} \tilde{e}$.[11] ■

This result requires additional explanation. The impact of changes in *e* over π^* has three different types of effects. The first effect is a *quantity effect*, which captures the impact of these changes over q^*. This effect is expressed by the first term of the right side of Eq. (15). We proved that when there is an increase (decrease) in *e*, the quantity of drugs sold by

[11] Recall that $\bar{e} = \frac{(1+d)^2}{(1+2d)}$.

an individual seller tends to decrease (increase). Second, a *price effect* that shows the impact of changes in *e* on equilibrium prices. Using Proposition 4, we verified that an increase (decrease) in *e* causes an increase (decrease) in P^*. This second effect occurs in the opposite direction of the first effect and is captured by the first part of the second term of the right side of Eq. (15), namely, the expression $q^*(n,e,d) \cdot \frac{\partial p^*(n,e,d)}{\partial e}$. Third, a *cost effect* reflects the impact of changes in *e* on the enforcement costs faced by a trafficker. An increase (decrease) in *e* produces an increase (a decrease) in these costs. Hence, this effect occurs in the same direction as the first effect does but is always smaller in magnitude than the second effect is.

Therefore, the net effect of a change in *e* depends on the size of the three types of described effects. When *e* is sufficiently large $(e > \tilde{e})$, the negative *quantity effect* is sufficiently strong to outweigh the positive sum of the *price effect* and the *cost effect* so that the profits decrease. Nonetheless, when *e* is relatively small $(e < \tilde{e})$, the positive *price effect* is sufficiently strong to outweigh the *cost effect* and the *quantity effect* simultaneously so that the profits increase. Hence, for a relatively low level of the *law enforcement intensity* $(e < \tilde{e})$, a marginal tightening thereof will result in a paradoxical positive net effect over π^*. In such cases, a repression policy is counterproductive, even when the number of traffickers and the network's density remain constant. It follows that repression policies can be effective only when they are strong. Otherwise, these policies induce decreases in the quantity of drugs sold while increasing the trafficker's profits, meaning that the sellers have incentives to produce more illicit drugs, even when they produce less at a given time.

However, for very small networks, i.e., when $n \leq 3$, or small networks with low average density levels, i.e., for $1 < d \leq 2$ and $3 < n \leq 5$, the profits are decreasing in *e*. The following propositions state this result formally.

Proposition 7: *Assume the following:* $1 < d \leq n-1$; $e < \bar{e}$; *and* $n \leq 3$. *Then, given a number of traffickers (n) and the network's average degree (d), an increase (decrease) in the law enforcement intensity (e) causes a decrease (an increase) in the trafficker's equilibrium profits.*

Proof: *Straightforwardly from Proposition 6 evaluating for* $n \leq 3$. ■

Proposition 8: *Assume the following:* $1 < d \leq 2$; $e < \bar{e}$; *and* $3 < n \leq 5$. *Then, given a number of traffickers (n) and the network's average degree (d), an increase (decrease) in* *the law enforcement intensity (e) causes a decrease (an increase) in the trafficker's equilibrium profits.*

***Proof*: *Straightforwardly from Proposition 6 evaluating for* $1 < d \leq 2$ *and* $3 < n \leq 5$. ■

Our findings are consistent with what is been called the "tipping points theory", for which the enforcement ability of the governmental authorities can engender different equilibrium points that can be extremely sensitive to changes in the intensity of repression[12]. From this theoretical perspective, in a "thin" illegal drug market, it is relatively easy to prevent the drug from spreading so that system can converge quickly to a low use-equilibrium with a moderate level of repression. Nonetheless, for a "large" market, it is difficult to restrict the expansion of illegal activities even with high levels of law enforcement (Caulkins and Reuter, 2010).

In our analysis, we determined three different possible states of the system:

1) For very small networks or small and very low-density networks, the trafficker's profits decrease with the intensification of repression.

2) For low-density networks and intermediate levels of the law enforcement intensity, profits increase with marginal increases in *e*.

3) For high levels of repression and intermediate or large networks with moderate or high levels of average density, profits decrease with marginal increases in *e*.

Consequently, there are three distinct "tipping points" in the system.

The net effect of positive changes in *n* over the profits is always negative. The following proposition states this effect this formally:

Proposition 9: *Assume that* $0 \leq d \leq n-1$ and that $e < \bar{e}$. *Then, given a law enforcement intensity (e) and the network's average degree (d), one unitary increase in the number of traffickers causes always a decrease (an increase) in the trafficker's equilibrium profits.*

***Proof*: *From Eq. (14), the rate of change of* π^* *with respect to one unitary change in n gives*

[12] See Kleiman (1988), Baveja *et al.* (1993), Caulkins (1993), Trager *et al.* (2001) and Caulkins and Reuter (2010).

$$\pi^*(n+1,e,d)-\pi^*(n,e,d)=\frac{1}{(n+1)}\cdot\left[1-\frac{e(1+2d)}{(d+1)^2}\right]\cdot\left\{-\left(1-\frac{e}{(1+d)}\right)+\left[1-\frac{e(1+2d)}{(d+1)^2}\right]\left(\frac{n(n+1)-1}{(n+1)(n+1)^2}\right)\right\} \tag{16}$$

This expression is negative for all n. ■

Proposition 9 shows that an increase in the number of traffickers always reduces the traffickers' profits because it induces two different types of effects on the profits: a *quantity effect* that captures the impact of the change of *n* over q_i^* and a *price effect* reflecting the impact over prices. Both effects are negative because of enhanced competition in the market.

The net effect of changes in *d* over the profits depends on the concrete value of *d*. The rate of change of π^* with respect to one unitary change in *d* gives

$$\pi^*(n,e,d+1)-\pi^*(n,e,d)=\frac{1}{n+1}\left[1-\frac{e(1+2(d+1))}{(d+2)^2}\right]\left\{\left[1-\frac{n}{(n+1)}\cdot\left[1-\frac{e(1+2(d+1))}{(d+2)^2}\right]\right]-\frac{e}{2+d}\right\}$$
$$-\frac{1}{(n+1)}\cdot\left[1-\frac{e(1+2d)}{(d+1)^2}\right]\cdot\left\{1-\frac{n}{(n+1)}\cdot\left[1-\frac{e(1+2d)}{(d+1)^2}\right]-\frac{e}{1+d}\right\}. \tag{17}$$

As in the case of a change in *e*, one variation in *n* causes three different types of effects that impact the profits: a *quantity effect*, a *price effect*, and a *cost effect.* While the first type of *effect* is positive, the second one is negative, as formulated by Propositions 3 and 5, respectively. The third one shows the negative influence of one change in *d* over the law enforcement costs. Hence, it represents a positive impact on profits, too. It can be proved that for relatively low (high) levels of *d*, one unitary increase of this parameter will result in an increase (a decrease) of the profits. The next proposition states this result formally.

Proposition 10: *Assume that* $0 \le d \le n-1$ and that $e < \bar{e}$. *Then, given a number of traffickers (n) and a law enforcement intensity (e), one unitary increase (a decrease) in the average density of the network, causes an increase in the trafficker's equilibrium profits for relatively small (large) values of d.*

Proof**:** *This proposition is difficult to proof analytically. However, simulating the model for different values of the parameters, we obtain this result straightforwardly.* ■

When the average network density is low, an increase in itself might lead to an increase in the seller's profits. Conversely, when it is relatively high an increase in itself leads to a decrease in the seller's profits. The reason for this hangs on the differences in the relative sizes of the *quantity* and *price effects* caused by the changes in the network's density. If the average network density is sufficiently high, then a marginal increase in itself leads to fall in the prices so strong as to outweigh the induced increase in the quantity of drugs sold as well as the provoked negative *cost effect*; the opposite happens for relatively low levels of the average network density.

5. The solution of the model in two special cases

It is important to consider two special cases in the regular networks setting: first, the case of a complete network, and second, the case of isolated traffickers.

Complete networks

Let us consider the case of a *complete network*, in which all the traffickers are linked to each other. Here, the average degree of the network, *d*, is $n-1$, so the average density is the highest possible value: *1*. From Eq. (9), in this special case, we obtain

$$q^*\left(n,e,\mathbf{g}^c\right)=\frac{n^2-e(2n-1)}{n^2(n+1)}; \tag{18}$$

Hence, the aggregate quantity of drugs sold is

$$Q^*\left(n,e,\mathbf{g}^c\right)=\frac{n^2-e(2n-1)}{n(n+1)}. \tag{19}$$

This outcome is the solution for the aggregate quantity of drugs sold in Poret and Téjedos's model, considering that we assume a simplified lineal demand with *a=b=1*[13] The equilibrium price gives

$$P^*\left(n,e,\mathbf{g}^c\right)=1-\frac{n}{(n+1)}\cdot\left[1-\frac{e(2n-1)}{n^2}\right] \tag{20}$$

Finally, the equilibrium profits of a seller are

$$\pi^*\left(n,e,\mathbf{g}^\mathbf{c}\right)=q^*\left(e,n,\mathbf{g}^c\right)\left(P^*\left(e,n,g^c\right)-\frac{e}{n}\right). \tag{21}$$

Using Eqs. (19) and (20), we obtain:

$$\pi^*\left(n,e,\mathbf{g}^c\right)=\left[\frac{n^2-e(2n-1)}{n^2(n+1)}\right]\cdot\left\{1-\frac{n}{(n+1)}\cdot\left[1-\frac{e(2n-1)}{n^2}\right]-\frac{e}{n}\right\} \tag{22}$$

We can denote this particular equilibrium levels for *q*, *Q*, *P* and π in the case of a complete network simply as q^c, Q^c, P^c and π^c respectively. This solution profile corresponds to the solution of Poret and Téjedo's model without considering the endogenisation of the horizontal structure of the market. It can be verified that within the assumptions of the model $q^* \leq q^c$. The next proposition states this formally

Proposition 11: *Assume that* $0 \leq d \leq n-1$ and that $e < \bar{e}$. *Then, the quantity of drugs sold by an individual seller in a complete network is always higher than in any other type of uniform network.*

---

[13] Recall that Poret and Téjedo denote the aggregate quantity of drugs sold as *q*.

Proof: Using *Eq. (9) and Eq. (19) we can observe that for constant values of e and n* $q^* \underset{<}{\geq} q^c \Leftrightarrow \frac{(1+2d)}{(d+1)^2} \underset{>}{\leq} \frac{(2n-1)}{n^2}$. *As* $\frac{(1+2d)}{(d+1)} \geq \frac{(2n-1)}{n^2}$, we have $q^* \leq q^c$. ■

It follows straightforward that for all $0 \leq d \leq n-1$ and $e < \bar{e}$, $Q^* \leq Q^c$, *and* $P^* \leq P^c$, for constant values of *e* and *n*. These results confirm that the more connected and denser is a network, the lower the equilibrium price and the higher the quantity of drugs sold both by an individual trafficker and in the entire market is, with the entire network being one extreme case of larger levels of drug production. Analogously, it can be proved that for all $0 \leq d \leq n-1$, and with $e < \bar{e}$, $\frac{\partial Q^*(n,e,d)}{\partial e} \leq \frac{\partial Q^c(n,e,\mathbf{g}^c)}{\partial e}$ for the same values of *e* and *n* in both cases. For this reason, in the complete network, we obtain the lowest marginal efficiency of the law enforcement policy. This result is consistent with the fact that $\frac{\Delta}{\Delta d}\left(\frac{\partial Q^*(n,e,d)}{\partial e}\right) > 0$, which indicates that the marginal efficiency of the law enforcement policy lowers with *d*.

Empty networks

Now, we consider the case with isolated agents with no links between one other. We noticed before that in the other special case, the probability of detection and arrest for a seller is always *h* and that the marginal law enforcement cost equals *e*. From (9), in the symmetrical Nash equilibrium, we obtain

$$q^*(n,e,\mathbf{g}^e) = \frac{1-e}{n+1}; \tag{23}$$

Hence,

$$Q^*(n,e,\mathbf{g}^e) = \frac{n(1-e)}{(n+1)}, \tag{24}$$

$$P^*(n,e,\mathbf{g}^e) = 1 - \frac{n(1-e)}{(n+1)}, \tag{25}$$

and finally,

$$\pi^*(n,e,\mathbf{g}^e) = \frac{(1-e)^2}{(n+1)^2}. \tag{26}$$

Let us denote these equilibrium levels as q^e, Q^e, P^e and π^e, respectively. We can demonstrate that, with our assumptions, $q^e < q^*$. The following proposition proves this inequality.

Proposition 12: *Assume that* $0 \le d \le n-1$ and that $e < \bar{e}$. *Then, the quantity of drugs sold by an isolated seller in a complete network is always lower than in any other type of uniform network.*

Proof: Using *Eq. (9) and Eq. (23), we can observe that for constant values of e and n* $q^e \overset{\ge}{\underset{<}{}} q^* \Leftrightarrow (d+1)^2 \overset{\le}{\underset{>}{}} (1+2d)$. *As* $(d+1)^2 \ge (1+2d)$, $q^e \le q^*$. ■

It is straightforward to prove that for all $0 \le d \le n-1$, and for all $e < \bar{e}$, $Q^e \le Q^*$, *and* $P^e \ge P^*$, for constant values of *e* and *n*. The direction of this finding is identical to that of the latter finding: in the other extreme case with isolated agents, with *e* and *n* being the same, we obtain the lowest production of drugs, both for individual sellers and for the entire market, and the highest equilibrium prices, thereby verifying the influence of the network's average density and hence, of its structure in the equilibrium levels. Additionally, it can be verified that for all $0 \le d \le n-1$, and for all $e < \bar{e}$, $\frac{\partial Q^e(n,e)}{\partial e} \le \frac{Q^*(n,e,d)}{\partial e}$, for identical values of *e* and *n*. This result is consistent with $\frac{\Delta}{\Delta d}\left(\frac{\partial Q^*(n,e,d)}{\partial e}\right) > 0$ too. Finally, combining these results, we can establish the following proposition:

Proposition 13:

Assume that $0 \le d \le n-1$ and that $e < \bar{e}$. *Then,* $q^e \le q^* \le q^c$.

Proof: *It follows by combining Propositions 11 and 12.* ■

This result implies that for a certain number of traffickers, the individual sales in the case of complete networks (*Poret and Téjedo's case* compared with our uniform-networks-setting), both individual sales and aggregate sales in the market are at their highest possible values, and the market equilibrium prices are their lowest possible values, while the *marginal* efficiency of the repression policies is at its lowest possible value.

The latter can be verified by noting that $\frac{\partial Q^e(n,e)}{\partial e} \le \frac{Q^*(n,e,d)}{\partial e} \le \frac{Q^c(n,e,d)}{\partial e}$.

The model with uniform networks elucidates the influence of the network's average density in the equilibrium levels of the quantity of drugs sold, their prices and the seller's profits. Although, it doesn't represent the general solution of the model and don't allows us to understand the impact of the strategic differences in the traffickers due to the network's structure, it captures the main aspects of the network's influence in the stylised context of a model with representative agents, and hence allows us to understand the main influences of the network's structure in the aggregate equilibrium. For this reason the model with uniform networks is useful to explain the real behaviour of the hard illicit drug markets as a whole. This is the purpose of the next section.

6. Understanding the failure of the war against drugs

The ineffectiveness of the supply repression policies to break off the reproduction and expansion of the drug trafficking business around the world during the past three decades, particularly until the first half of the past decade, can be explained by the structural change exhibited by the drug trafficking and distribution networks during this period. As the drug trafficking organisations evolved in strategic response to the shifting historical conditions and to the progressive tightening of the repression policies applied by law enforcement authorities over this period, the criminal networks supporting them evolve in different and specific ways and magnitudes, depending on the concrete place and moment at which they performed their criminal activities, in all cases fuelling expansion around the world.

In fact, during the nineties and the first half of the following decade, the incidence of illegal activities rose overall, while the criminal networks supporting them expanded and became denser in certain sub-structures and local entities of the illegal organisations. Some of the emerging drug trafficking organisations began to control the most important international drug trafficking distribution networks at a wholesale level, which had been controlled, in the preceding era, by the large cartels such as the Cali Cartel and the Medellín Cartel.

Two key factors account for these changes. On the one hand, the dismantling of the large cartels during the mid-nineties marked the end of the “narco-terrorism era”: an age characterised by the operation of big cartels with hierarchical structures. A new historical era of drug trafficking began at this time that can be characterised by the emergence of numerous small and atomised cartels, which tended to operate in connection to transnational organisations to conduct their trafficking and transportation activities more efficiently. These new small cartels, sometimes called “cartelitos”, however, constituted high-dense networks characterised by clustered-hierarchy structures. Due to their small size and their consequent weak defence and military capability, they could not sustain their own defence and security forces, so they were compelled to hire the defence and security forces of large illegal armed groups, such as the AUC paramilitary armies and the FARC guerrilla, with the aim of consolidating *defence systems* for their service. Hence, the control exerted by the large illegal armed groups over great expanses of land enabled the expansion of the illegal coca crops as well as opium poppy crops, but for a shorter period and to a lesser degree for the latter, across vast territories in Colombia.

On the other hand, the nineties witnessed the *balloon effect* on the *Andean Region*[14]. Owing to this phenomenon, across the entire territory of the main cocaine producers of the

[14] *The balloon effect* refers to the process of geographical reallocation of the illegal crops ensuing from the implementation of repression policies in certain zones. Owing to the incidence of the *balloon effect,* historically, the diminishing of illegal crops in some countries or regions has contrasted with their increase in other places. The impact of the *balloon effect* has been supported by several works in the field (see, for example, Ortiz (2009), Caicedo (2006 and Rouse and Arce (2006)).

continent (Colombia, Perú and Bolivia), the efficacy of law enforcement policies, i.e., the eradication policies, to reduce the illegal crops in some areas contrasted with their growth in another ones at the same time.

In summary, the outcome of *balloon effects* on the coca crops as well as on the performance of trafficking activities has effectively facilitated the *resilience* of criminal organisations at both national and transnational levels, while thus availing these organising with tremendous capabilities to quickly reallocate economic activities for the entire value chain at any links when urgently required. The *resilience* of the criminal organisations is one of their basic properties, and it is worth understanding their rapid expansion during the last few decades, despite strong control exerted in the *war against drugs* during all of these years.

Furthermore, our model shows that though a marginal tightening in the repression intensity, *ceteris paribus*, always decreases drug sales, it can, at the same time, increase traffickers' profits, thus encouraging them to increase their sales in the future despite any present reductions in profit. Based on our model, this phenomenon can be observed in low-density networks and in cases with intermediate levels of law enforcement intensity. This result has likely been the recurrent state of affairs since the nineties at a transnational level. Since that time, the illegal organisations have typically operated by holding high-clustered networks at a local level in production and local distribution networks, while displaying low density transport and trafficking networks at a transnational level. Given this operational protocol, it has been difficult for law enforcement authorities to apprehend the concrete level of repression that could actually block the performance of illegal activities under specific historical circumstances. Mostly, a level of repression may be required that cannot be practically attained because of its high fiscal and social costs.

Our findings are consistent with the main findings of Poret and Téjedo (2006). They show strong evidence of a reorganisation of the production and transport of cocaine that have occurred since the nineties and that have led to the emergence of a great number of new and more flexible illegal organisations of drug trafficking and distribution. In their model, this process is captured as a progressive increase in the number of traffickers, a result that actually indicates a transformation of the illegal market structures to more competitive markets.

In our networks-setting model, an increase in the number of traffickers concomitantly signals an expansion of social networks, i.e., the social structure of the delinquent activities related to drug trafficking, supporting the illegal market structures. An increase in the average density of the drug trafficking networks can be conceptualised as another type of transformation of the illegal market structures, which also encourage competition, as previously demonstrated.

To summarise, our model helps to advance research towards explaining the failure of the supply repression policies to halt the reproduction of the illegal activities. In particular, it reveals that larger number of traffickers and/or higher levels of density of the drug trafficking networks in which they are embedded precipitate increases in illegal drug sales as well as in declines in psychotropic substance prices. In real world both phenomena, a growth in the number of traffickers and a densification of the drug trafficking networks can arise concurrently (Caulkins and Reuter, 2010). Hence, considering the actual operations of more robust networks and larger markets that overlap with each other provides a subtle resource for understanding how illegal activities have been operating and have been evolving over the last few decades, despite strong repression exerted by the control authorities in the context of *the war against drugs*.

Concluding remarks

Our model notes the relevance of the role of drug trafficking and distribution network structure for understanding how illegal activities operate and evolve. For the case of drug trafficking, our findings corroborate the hypothesis of Calvó-Armengol and Zenou (2004) and Ballester *et al.* (2006, 2009), whereby delinquent activities enhance their performance as the criminal networks supporting them expand. Specifically, larger and/or denser networks allow the commitment of higher delinquent efforts, in our case, a higher level of aggregate sales of illicit drugs. Therefore, our model highlights the fact that the expansion and the densification of drug trafficking networks are key trends for explaining the permanent growth of the drug sales and the consequent decline in prices that have occurred during the past three decades.

In the context of our analytical framework, we proved the following findings by solving the model for the stylised case with uniform networks. We showed that Poret and Téjedo were correct in that the aggregate quantity of drugs sold depends negatively on the intensity of the law enforcement policies applied but depends positively on the number of traffickers in the market. Indeed, we verified that whereas the aggregate sales in the market are a lineal-decreasing function of the intensity of the law enforcement policies, they are a concave-increasing function of the number of traffickers in the market.

Second and given the first result, we proved that the equilibrium market prices of illicit drugs follow a positive lineal function of the level of the law enforcement intensity and a convex-decreasing function of the number of traffickers in the market.

Third, we proved that the individual and aggregate sales in the market depend positively on the network’s average density (i.e., the network’s average degree). This proof is one of the original contributions of this paper. The intuition behind this result is that the greater the number of social connections each trafficker has, the higher the quantity of drugs sold by him, for the reason that with a greater number of links, each seller faces more positive network externalities in relation to his costs and thus has more of an incentive to increase his sales to obtain higher profits.

Fourth and following from the third result, we showed that the equilibrium market prices of illicit drugs are a convex-decreasing function of the average density of the networks. This logic is one of the main reasons for the decline exhibited by the hard drug prices in the long-run during the past decades. The other reason is the increased number of traffickers in the market.

We also obtained noteworthy results regarding the behaviour of the equilibrium profits of the traffickers. We found that depending on the structure of the drug trafficking network, a rise in the law enforcement intensity can increase or decrease the profits of the individual traffickers. This result is consistent with the "tipping points theory" of illegal markets developed by Caulkins and others. This theory asserts that the enforcement ability of the governmental authorities can create different equilibrium points that can be extremely sensitive to changes in the intensity of repression. In our analysis, we found three possible states of the system related to the impact of law enforcement policies on trafficker's profits. 1) For very small networks or small and very low-density networks, the trafficker's profits decline as repression intensifies. 2) For low-density networks and intermediate levels of law enforcement intensity, profits increase with marginal increases in *e*. 3) For high levels of repression and intermediate or large networks with moderate or high levels of average density, profits decrease with marginal increases in *e*.

Additionally, we found that an increase (decrease) in the number of traffickers in the market causes a decrease (an increase) in the trafficker's equilibrium profits. This effect is explained by the negative impact on market equilibrium prices and individual sales that ensues with increased seller competition within the market.

Furthermore, it can be proved that for relatively low levels of *d*, one unitary increase of this parameter will result in an increase in profits. Conversely, when *d* is relatively high, an increase in its value leads to a decrease in the seller's profits. The reason for this result is based on the differences in the relative sizes of the *quantity* and *price effects* caused by changes in the network density.

Finally, we found that the *marginal efficiency* of the law enforcement policies depends on the network's density level. This efficiency dampens as network density increases. We showed that Poret and Téjedo's model corresponds to one special case of our networks-setting model in which networks are complete. Hence, the actual *marginal efficiency* of the law enforcement policy could be eventually more effective, *ceteris paribus*, than in this special case.

Nonetheless, as we have explained, the actual effectiveness of law enforcement policies depends on the levels of the other parameters, particularly on the number of traffickers (*n*) and the average density of the networks (captured by the level of the network's average degree, *d*). That is, it depends exclusively on the network structure and the latter's evolution.

References

Ballester, C., A. Calvó-Armengol and Y. Zenou (2009). "Delinquent Networks", *IZA Discussion Paper Series* 4112, pp. 1-43.

Ballester, C., A. Calvó-Armengol, and Y. Zenou (2006). "Who's Who in Networks. Wanted: The Key Player", *Econometrica* 74, 5, pp. 1403-1417.

Baveja, A., R. Batta, J. P. Caulkins, and M. H. Karwan (1993). "Modeling the Response of Illicit Drug Markets to Local Enforcement", *Socio-economic Planning Sciences* 27(2), pp.73–89.

Becker, G. S., K. M. Murphy and M. Grossman (2006). "The Market for Illegal Goods: The Case of Drugs", *Journal of Political Economy* 114, 1, pp. 38-60.

Burrus, R. T. (1999). Do Efforts to Reduce the Supply of Illicit Drugs Increase Turf War Violence? A Theoretical Analysis, *Journal of Economics and Finance* 23, pp. 226– 234.

Caicedo, C. J. (2006). "Expectativas Racionales y Persistencia de los Cultivos Ilícitos de Coca en Colombia 1986-2003", CIDER, Universidad de los Andes, Colombia.

Calvó-Armengol, A. and Y. Zenou (2004). "Social Networks and Crime Decisions: The Role of Social Structure in Facilitating Delinquent Behavior", *International Economic Review* 45, 3, pp. 939-958.

Caulkins, J. P. and R. Padman (1993). "Quantity Discounts and Quality Premia for Illicit Drugs", *Journal of American statistical Association* 88, pp. 748-757.

Caulkins, J. P. and P. Reuter. (2010). "How Drug Enforcement Affects Drug Prices?", *Crime and Justice* 39, 1, 2010, pp. 213-271.

Hagedorn, J. *People and Folks: Gangs, Crime and the Underclass in a Rustbelt City* (1988). Chicago: Lake View Press.

Kleiman, M. A. R. (1988). "Crackdowns: The Effects of Intensive Enforcement on Retail Heroin Dealing". In *Street-Level Drug Enforcement: Examining the Issues*, edited by Marcia R. Chaiken. Washington DC: National Institute of Justice.

Miron J (2001). "The Effect of Drug Prohibition on Drug Prices: Theory and Evidence", *Working Paper*. Boston University y Bastiat Institute.

Ortiz, C. H. (2002). "Luchando infructuosamente contra la hidra: Un modelo sencillo del narcotráfico", *Cuadernos de Economía* 37.

Ortiz, C. H. (2003). "La guerra contra las drogas es contraproducente: un análisis económico de equilibrio general", *Lecturas de Economía* 58, pp. 47-68.

Ortiz, C. H. (2009). "The War on Drugs is Counterproductive, Once Again", *Revista Lecturas de Economía* 71, pp. 19-42.

Padilla, F. (1992). *The Gang as an American Enterprise.* New Brunswick, NJ: Rutgers University Press.

Poret, S. (2002). “Paradoxical Effects of Law Enforcement Policies: The Case of the Illicit Drug Market”, *International Review of Law and Economics*, 22, pp.465-493.

Poret, S. and C. Téjedo (2006). “Law Enforcement and Concentration in Illicit Drug Markets”, *European Journal of Political Economy* 22, pp. 99-114.

Raffo, L. (2010). “Narcotráfico y conflicto: Por qué bajó el precio de la cocaína”, *Revista de Economía Institucional* 12, 23, pp. 229-258.

Raffo, L. (in press). “La baja de largo plazo en los precios de las drogas duras: una explicación”, *Lecturas de Economía*, No 83, julio-diciembre de 2015.

Rouse, S., y M. Arce (2006). “The Drug-Laden Balloon: US Military Assistance and Coca Production in the Central Andes”, *Social Science Quarterly* 87, 3, pp. 540-557.

Sutherland, E.H. (1947). *Principles of Criminology*, fourth edition, Chicago: J. B. Lippincott.

Thornberry, T.P., M.D. Krohn, A.J. Lizotte, and D. Chard-Wierschem (1993). “The Role of Juvenile Gangs in Facilitating Delinquent Behavior,” *Journal of Research in Crime and Delinquency* 30, pp. 55-87.

Tragler, G., J. P. Caulkins, and G. Feichtinger (2001). “Optimal Dynamic Allocation of Treatment and Enforcement in Illicit Drug Control”, *Operations Research* 49(3), pp. 352–62.

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